SU(7) GRAND UNIFIED THEORY

KATSUJI YAMAMOTO

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KATSUJI YAMAMOTO

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Contents

§1. Introduction: The Genealogy of Unification of Elementary Interactions

In the 1970's we met with a great and substantial change in the picture of elementary particles: *"From Hadrons to Quarks"*. Some people became already in the 1960's aware of the fact that hadrons and their interactions can be classified very suitably by supposing more fundamental ingredients, *"quarks"*, of which hadrons are composed.[1),2)] However, they regarded quarks merely as a mathematical tool, i.e., as a "working hypothesis" to realize the unitary symmetry among hadrons, since in those days there were not any compelling experimental facts to indicate the reality of quarks.

The discovery of the vector meson J/ψ in November, 1974[3)] made the "Quark Revolution" decisive. The J/ψ was found to have a very long lifetime in spite of its heavy mass (3.1GeV) in contrast with other known heavy hadrons. Thus, J/ψ could not be understood in the old SU(3) scheme realized by three kinds of quarks (u,d,s). A natural explanation of this curious property of J/ψ was given by introducing a new kind of quark (c) with a quantum number "charm" and considering that J/ψ is a *dynamical bound state of* $\bar{c}c$. Meanwhile, vector mesons ψ' and ψ'' which were regarded as radial excited states of J/ψ were discovered.[4)] Their mass spectra also fitted rather well in the nonrelativistic calculation for the $\bar{c}c$ system.[5)] These circumstances show that the $\bar{c}c$ quark-antiquark pair is not a mere mathematical symbol but a physical object. Furthermore, the recent e^+e^- annihilation experiments have brought us a clever method to see quarks "semi-directly" through hadron jets, i.e., we can suppose that a prompt quark and antiquark are primarily produced, then they break into fragments to produce hadron jets in such a way as $e^+e^- \to q\bar{q} \to$ two jets.[6)]

The physical consensus brought by the "Quark Revolution" may be expressed as follows:

(i) Quarks are not mathematical symbols but *real* objects even though they may not be seen as free particles.

(ii) *Quarks and leptons* are indeed fundamental building blocks of matter at the level of our present knowledge.

(iii) As is explained in detail below, in the quark-lepton point of view, the strong, electromagnetic, weak and gravitational interactions[*)] are all described by *gauge theories*[8)] in which interactions are mediated by the gauge bosons, in contrast with the fact that at the hadron level these interactions are treated in completely different fashions each

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*) We do not consider the gravitational interactions hereafter since they are much much weaker than the others. However, they may play an important role in the ultimate understanding of nature. The supergravity theory[7)] is one of the interesting attempts at unification of all interactions including the gravity.

other. *So it should be emphasized that the way toward the unification of interactions had not been opened until we rushed into the "Quark-Lepton World".*

(iv) Therefore, the study of the *"Quark-Lepton World"* has become the main avenue of particle physics.

Until now, six leptons $(e,\mu,\tau,\nu_e,\nu_\mu,\nu_\tau)$ and five quarks (u,d,s,c,b) have been known, and the sixth quark (t) is to appear from the quark-lepton correspondence. This variety of quarks and leptons is called *"flavour"*. These quarks and leptons are divided into two types, respectively, according to their electric charges:

up-type quarks	(u,c,t)	$Q = 2/3$,
down-type quarks	(d,s,b)	$Q = -1/3$,
charged leptons	(e,μ,τ)	$Q = -1$,
neutrinos	(ν_e,ν_μ,ν_τ)	$Q = 0$.

The left-handed components of up-type and down-type quarks, and those of neutrinos and charged leptons, respectively are combined into doublets with respect to the weak interactions, and all the right-handed components are singlets (if there are right-handed neutrinos):

$$\begin{array}{ccccccl}
\begin{pmatrix} u \\ d \end{pmatrix}_L & ; & \begin{pmatrix} c \\ s \end{pmatrix}_L & ; & \begin{pmatrix} t \\ b \end{pmatrix}_L & \multirow{2}{*}{\Bigg\}} & \text{quarks,} \\
u_R, d_R & ; & c_R, s_R & ; & t_R, b_R & & \\
\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L & ; & \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L & ; & \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L & & \text{leptons,} \\
e_R, \nu_{eR} & ; & \mu_R, \nu_{\mu R} & ; & \tau_R, \nu_{\tau R} & & \\
\begin{array}{c}\text{1st}\\ \text{generation}\end{array} & ; & \begin{array}{c}\text{2nd}\\ \text{generation}\end{array} & ; & \begin{array}{c}\text{3rd}\\ \text{generation}\end{array} & , &
\end{array}$$

where the left-handed component ψ_L and the right-handed one ψ_R of a Dirac field ψ are defined by $\psi_L = \frac{1}{2}(1-\gamma_5)$, $\psi_R = \frac{1}{2}(1+\gamma_5)$, respectively. The groups (u,d,e,ν_e), (c,s,μ,ν_μ) and (t,b,τ,ν_τ) are called 1st, 2nd and 3rd generations, respectively. This arrangement reflects the fact that the charged weak decays, $\pi^+ \to \mu^+ + \nu_\mu$, $n \to p + e^- + \bar{\nu}_e$, etc., have V-A and SU(2) structures,[9] that is, the space integral of the time component of the left-handed charged weak currents $\bar{\nu}_{eL}\gamma_\mu e_L$ etc. behaves as a generator of SU(2). On the other hand, it is well known that the electromagnetic interactions are described very successfully by a gauge theory based on an Abelian group $U(1)_{em}$.[10]

The first step was taken toward the unification of interactions, by marrying the electromagnetic interactions and the weak interactions into one gauge theory based on

$SU(2)_W \times U(1)$.[11] The local gauge symmetry $SU(2)_W \times U(1)$ is supposed to be spontaneously broken[12] into $U(1)_{em}$ at the mass scale about 100GeV so that the gauge bosons which mediate weak processes can be massive enough to make the weak interactions much weaker than the electromagnetic interactions in the low-energy region as is observed. When we go beyond 100GeV, the $SU(2)_W \times U(1)$ symmetry will be restored and the magnitude of the weak interactions will become of the same order as that of the electromagnetic interactions.

The $SU(2)_W \times U(1)$ model of electro-weak interactions (Glashow-Weinberg-Salam model), which was already proposed in the 1960's, predicted the existence of weak *neutral* currents which had not been observed yet in those days. After several years, the elastic neutrino scatterings ν_μ-e and ν_μ-p due to the weak neutral current interactions were discovered at CERN in 1973,[13] which proved that the $SU(2)_W \times U(1)$ model is plausible and attractive. By 1978, it has been confirmed that the observed neutral current processes induced by neutrinos[14] and the parity violation in polarized e^--deuteron scattering[15] are all explained quantitatively in terms of only two phenomenological parameters, G_F (Fermi constant) and $\sin^2\theta_W$ $=0.23 \pm 0.02$ (Weinberg angle), which are intrinsic in the $SU(2)_W \times U(1)$ theory. Therefore, we may conclude that the unified description of weak and electromagnetic interactions based on the $SU(2)_W \times U(1)$ gauge theory has been established at least in the low-energy region. There are of course some interesting extended models such as $SU(2)_L \times SU(2)_R \times U(1)$

left-right symmetric model[16] which reproduce the same results in the low-energy as the standard $SU(2)_W \times U(1)$ model does. It is left for the future experimental survey in the high-energy region (100GeV$\sim$1TeV) to decide which is a true electro-weak theory. The $SU(2)_W \times U(1)$ theory and its extensions are often called *quantum flavourdynamics (QFD)*.

Next, let us view the features of strong interactions. Quarks are thought to have, in addition to "*flavour*", an internal degree of freedom, "*colour*", which distinguishes them from leptons. Several theoretical and experimental facts indicate that the colour symmetry is $SU(3)_C$. With respect to the $SU(3)_C$, quarks are *tricolour*, while leptons are colourless. We can easily conjecture that the $SU(3)_C$ is a local gauge symmetry. Then, quarks interact strongly with each other through the exchange of $SU(3)_C$ gauge bosons, *gluons*. The gauge theory based on the colour $SU(3)_C$ is called *quantum chromodynamics (QCD)*.[17] Hadrons are colourless bound states of quarks and antiquarks. Quarks are supposed to be confined inside hadrons by the strong QCD force. The strong forces among hadrons such as nuclear force may be understood as molecular-force-like residual interactions of QCD.

QCD is a very hopeful and attractive theory for strong interactions. Especially, it has a remarkable feature, i.e., "asymptotic freedom".[18] The "asymptotic freedom" implies that the effective QCD coupling constant decrease logarithmically by renormalization effect as available momenta increase. The momentum dependence of the running gauge coupling constant is governed by a suitable renormalization equation. Thanks to the "asymptotically free" character of QCD, we can perform perturbative calculations for the deep inelastic scattering, the e^+e^- annihilation etc. at high-energy. Further, it should be added that the experimental evidence for QCD has been brought recently from the three jet phenomenon in e^+e^- annihilation, in which one of the three jets is interpreted as bremsstralung of a hard gluon, i.e., $e^+e^- \to q+\bar{q}+g$.[19)]

We can summarize the above story in the following way: At the present status of elementary particle physics the known interactions among quarks and leptons are governed by the "standard" theory;

$$(\mathrm{QCD})\times(\mathrm{QFD}) = SU(3)_C\times SU(2)_W\times U(1).$$

As is seen above, the "standard" theory is very successful. It is, however, still far from the final theory and many questions are left unexplained. Above all, it cannot be thought a complete unification since it has three independent gauge coupling constants g_C, g and g' associating with $SU(3)_C$, $SU(2)_W$ and $U(1)$, respectively.

Thus we have been led to the idea of the *grand unified theory (GUT)* to unify the $SU(3)_C\times SU(2)_W\times U(1)$ into a *simple* or *semisimple* group with *one* gauge coupling constant.[20)∿22)]

The genealogy of unification to reach GUT is illustrated as follows:

$$
\left.
\begin{array}{l}
\left.\begin{array}{l}\text{electromagnetic}\\[2ex] \text{weak}\end{array}\right]\longrightarrow \begin{array}{c}SU(2)_W\times U(1)\\ \text{(QFD)}\end{array}\\[6ex]
\text{strong} \longrightarrow \begin{array}{c}SU(3)_C\\ \text{(QCD)}\end{array}
\end{array}
\right] \longrightarrow \text{GUT} .
$$

Along this line of thought, Georgi and Glashow proposed an SU(5) grand unified model.[21] In the SU(5), each generation of fermions are assigned to a reducible representation ($\mathbf{5}^*+\mathbf{10}$) of SU(5):

$$
\begin{array}{c}\mathbf{5}^*\\ \begin{pmatrix} \nu_e & d^c \\ e^- & \end{pmatrix}_L\end{array} , \quad
\begin{array}{c}\mathbf{10}\\ \begin{pmatrix} & u & \\ e^+ & & u^c \\ & d & \end{pmatrix}_L\end{array} \quad \text{etc.}
$$

The SU(5) GUT brought us some admirable outcomes:

(i) The Weinberg angle is predicted to be $\sin^2\theta_W=3/8$ ($g/g'=\tan\theta_W$).

(ii) The electric charge is quantized, i.e., this fact explains why the charge unit of quarks is 1/3, or more specifically why the charge of proton = - the charge of electron.

(iii) Protons decay due to new interactions $SU(5)/SU(3)\times SU(2)\times U(1)$.

The properties (i) and (ii) result from the fact that the generators of $SU(3)_C$, $SU(2)_W$ and U(1) are normalized in the same fashion to those of SU(5). The meta stability of proton ($\gtrsim 10^{30}$ years)[23] requires that the SU(5) should be spontaneously broken into the $SU(3)_C \times SU(2)_W \times U(1)$ at super high energy ($\gtrsim 10^{15}$GeV) so that the gauge bosons in the $SU(5)/SU(3)\times SU(2)\times U(1)$ should be superheavy. It should be mentioned that the above features are common in all GUT's.

Here, important questions to the SU(5) (or generally to GUT) are posed: Why is the strong interaction really stronger by about one order of magnitude than that of the electro-weak interaction, in spite of the fact that both of them come from the same origin of SU(5)? Why is the experimental value of $\sin^2\theta_W (0.23\pm 0.02)$ somewhat different from the canonical one (3/8)? An elegant answer for these questions was given by Georgi, Quinn and Weinberg.[24] They noticed that after the spontaneous breaking of the SU(5) the gauge coupling constants g_C, g and g' receive different renormalization effects each other due to the different gauge interactions $SU(3)_C$, $SU(2)_W$ and U(1), respectively, since superheavy particles decouple out of the game.[25] Then, they found by solving the renormalization equations for g_C, g and g' that the grand unification mass, at which the SU(5) symmetry is restored, should be as large as 10^{15}GeV to give rise to renormalization effects necessary for explaining the values, $\alpha\equiv e^2/4\pi=1/128$,*) $\alpha_s\equiv g_C^2/4\pi=0.1\sim0.2$ and $\sin^2\theta_W=0.23\pm0.02$ which are observed in the low-energy region ($\lesssim 10^2$GeV).[14),26),27)] In this way it is easily envisaged that g_C, g and $\sqrt{5/3}\,g'$ are all identical to the SU(5) gauge coupling constant at 10^{15}GeV (where $\sqrt{5/3}$ is the normalization factor for U(1)), while they depart from each other as energy decrease. The energy dependences of gauge coupling constants are displayed in Fig.1, where the strong coupling constant g_C shows "asymptotic freedom". The huge gap between ordinary mass (10^2GeV) and unification one (10^{15}GeV) is called "*hierarchy*" of interactions.

It is very impressive that the two independent estimates for the unification mass agree with each other which are obtained from essentially different requirements, i.e., the observed stability of proton and the unification of interactions (coincidence of g_C, g and $\sqrt{5/3}\,g'$ at unification mass). In other words, this means that the proton lifetime ($\sim10^{30}$ years) calculated from the unification mass ($\sim10^{15}$GeV) in the SU(5) is fortunately on the border of present experiments.[23)] Therefore, we can expect that the validity of GUT idea may be proved in the near future by the discovery of proton decay.

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*) Note that this value of the fine structure constant is slightly different from the Thomas limit (zero momentum transfer limit) one (1/137) since it is renormalized at 100GeV.

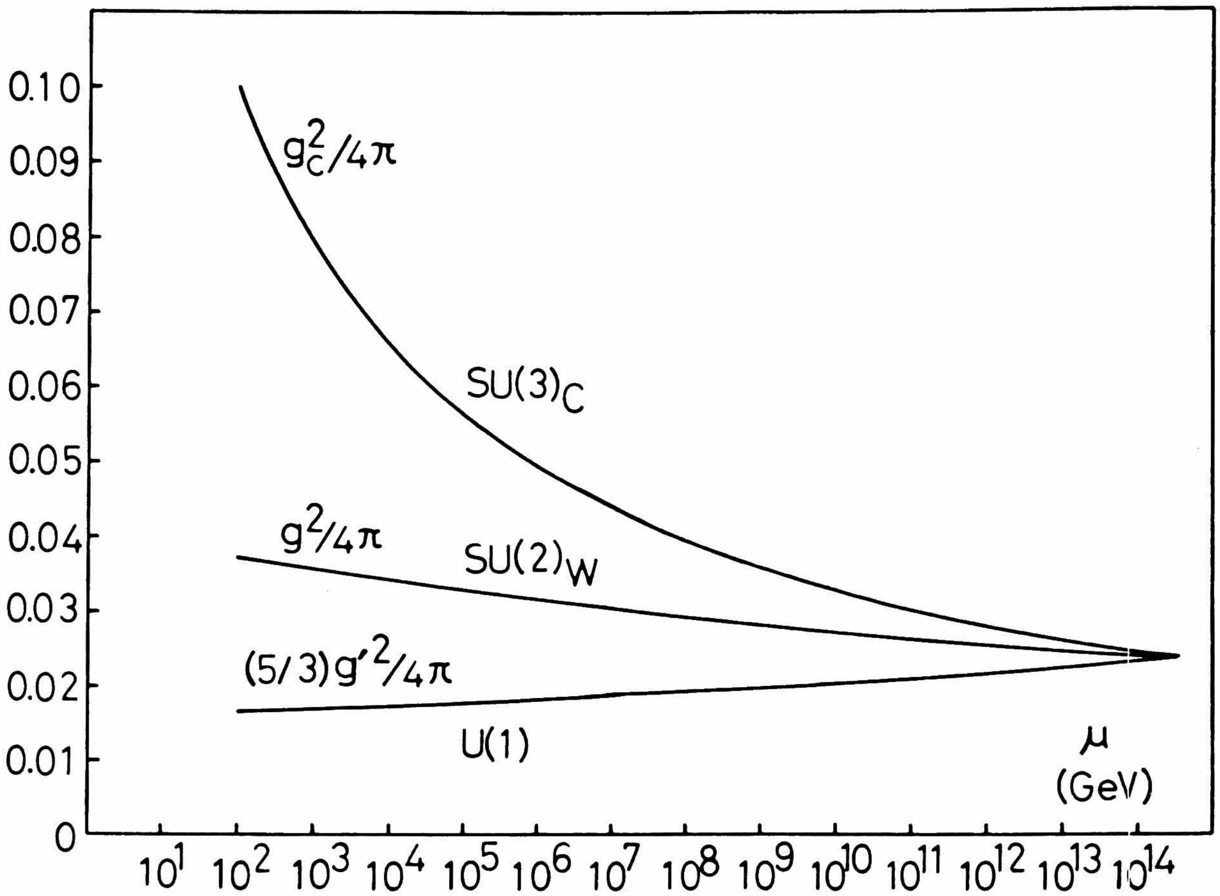

Fig.1. The energy dependences of gauge coupling constants in the SU(5).

As is well known, the SU(5) is a prototype of GUT's. It has, however, unpleasant features:

(i) The SU(5) is *one* generation theory. Each generation of fermions (u,d,ν_e,e) etc. belongs to the same representation $(\mathbf{5}^*+\mathbf{10})$ of SU(5). This is the up to date version of the old μ-e puzzle. *"Why does Nature repeat herself?"*

(ii) There is a huge "physical desert" between the ordinary mass scale (10^2GeV) and the unification one (10^{15}GeV) in the SU(5) since SU(3)×SU(2)×U(1) is a maximal subgroup of SU(5). So we cannot observe any other interactions than that of the "standard theory" to the GUT region. *"Can we come across any new interactions such as extended electro-weak interactions, $SU(3)_W\times U(1)$, in the feasible future?"*

Motivated by these questions, we are led to the extension to larger GUT groups than the SU(5), expecting possible new physics that we have never seen and searching for a clue to investigate the basic structure of nature.

In this thesis, we want to study interesting features of extended GUT's based on SU(N) gauge group, especially concentrating on the SU(7) GUT, from the above viewpoints.

The first subject in this work is to investigate possible structures of the gauge hierarchy of interactions for grand unification models (GUM's), especially for the SU(N) model. By considering renormalization effects on relevant gauge coupling constants à la Georgi, Quinn and Weinberg,[24] we will derive several sum rules, which are relations between α, α_s,

$\sin^2\theta_W$ and possible threshold masses M_i of new interactions. These sum rules reveal clearly and quantitatively correlations between the enlargement of the colour group and that of the flavour one. By applying these sum rules, we will analyze numerically possible steps of enlargement of interactions such as $SU(3)_C\times SU(2)_W\times U(1) \rightarrow SU(4)_C\times SU(3)_W\times U(1)$ and the corresponding threshold masses M_i. We will also discuss the modified SU(N) model with $\sin^2\theta_W\neq 3/8$ at the unification mass, which involves new quarks with charge $Q\neq n/3$ and/or new leptons with fractional charge.

The second subject is to give a detailed discussion on an SU(7) grand unified model with breaking pattern $SU(7) \rightarrow SU(4)_C \times SU(3)_W\times U(1) \rightarrow SU(3)_C\times SU(2)_W\times U(1) \rightarrow SU(3)_C\times U(1)_{em}$, which is taken through the first subject as a simple and attractive candidate of extended GUT and has an intermediate stage ($10^5 \sim 10^7$GeV) of new interactions $SU(4)_C\times SU(3)_W\times U(1)$. In our SU(7) GUM, the fundamental representation **7** consists of SU(5)'**5** and its two singlets with the opposite charges $\pm q$ ($q=1/2$). We will exhibit characteristic features of the SU(7), i.e., evasion of the survival hypothesis, suppression of left-handed neutrino masses and so on, deeply connected with the bold choice ($q=1/2$) for the electric charge assignment. Furthermore, we will show for completeness that the desired breaking pattern of the SU(7) is indeed realized and the positivity of $(\text{mass})^2$'s of physical scalars is surely guaranteed in a finite range of the coupling constants of the Higgs potentials under suitable massless conditions for Higgs scalars which cause the symmetry breaking at the successive stages.

The third subject is to investigate systematically various symmetry breaking patterns in the SU(7) unification and to discuss their implications. We can expect new interactions such as extended "colour" $SU(4)_C$, extended "flavour"(electroweak) $SU(3)_W \times U(1)$ and/or extra U(1)'s at each stage of symmetry breaking paths. We will study phenomenological aspects of these interactions and give the corresponding mass scales.

The final subject in this thesis is to study effects of terrestrial mirror fermions with V+A coupling for the $SU(2)_W \times U(1)$ which appear in a class of extended GUM's such as SU(7) or SO(14). We will find that flavour changing neutral currents associated with the Z boson of $SU(2)_W \times U(1)$ are induced by the slight mixing of ordinary and mirror fermions. Then, we will show that the effects on flavour changing processes such as the K_L-K_S mass difference are safely suppressed as far as "ordinary"-"mirror" mixings are sufficiently small ($\lesssim 10^{-2} \sim 10^{-3}$).

The contents of this thesis are divided into seven parts. In §2, possible structures of gauge hierarchies in the SU(N) unification are investigated. Next, an SU(7) grand unified model is presented and its attractive features are discussed in §3. The succeeding section (§4) is devoted to a detailed analysis for the Higgs potentials to realize a desired symmetry breaking pattern of the SU(7). In §5, various symmetry breaking patterns in the SU(7) unification and their implications are systematically investigated. In §6, effects of terrestrial mirror fermions on flavour changing neutral currents are studied. Summary and concluding remarks are presented in §7.

The author made this thesis by completing the works which he has studied up to present with I. Umemura. These works have been published in references 28), 29), 30) and 31).

§2. Possible Structures of Gauge Hierarchies in the SU(N) Unification

Possible structures of the gauge hierarchy of interactions are investigated for grand unification models, especially for the SU(N) model. Several sum rules are derived, which are relations between α, α_s, $\sin^2\theta_W$ and possible threshold masses M_i of new interactions. These sum rules reveal clearly and quantitatively correlations between the enlargement of the colour group and that of the flavour one. By applying these sum rules, possible steps of enlargement and the corresponding threshold masses M_i are analyzed numerically. The modified SU(N) model with $\sin^2\theta_W \neq 3/8$ at the unification mass is discussed also, which involves new quarks with charge $Q \neq n/2$ and/or new leptons with fractional charge.

§2.1 Introduction

Grand unified theories of the strong, weak and electromagnetic interactions[20)∿22)] can predict definite values of $\sin^2\theta_W$ and α/α_s at the unification mass scale, which are different from those observed at the available energy region. In order to understand these different values of $\sin^2\theta_W$ etc. as renormalization effects by gauge theories, a large scale gap between the unification mass and the ordinary one should be supposed. Along such a line of thought, Georgi, Quinn and

Weinberg (abbreviated to GQW hereafter) proposed an idea of 'hierarchy of interactions'.[24]

Recently, Dawson and Georgi (DG)[32] generalized the analysis of coupling-constant renormalization in unified gauge theories to allow for more than one scale of spontaneous symmetry breakdown between unification and $SU(3)_C\times SU(2)_W\times U(1)$. They applied their analysis to the SU(N) unification with SU(5) embedded in such a way that the N-dimensional representation consists of an SU(5) **5** and N-5 neutral singlets, and obtained the following relation:

$$\cos 2\theta_W/\alpha(M_W) - (2/3)/\alpha_s(M_W) \simeq (22/3\pi)\ln(M/M_W) \ ,$$
$$(\ M = \text{unification mass},\ M_W = \text{ordinary mass}\ )$$

which is independent of the details of the gauge hierarchy. This relation coincides with a linear combination of two relations which GQW first obtained without any intermediate hierarchies between the unification scale and the ordinary one. This suggests that in the SU(N) grand unification, the unification mass M estimated in terms of α, α_s and $\sin^2\theta_W$ renormalized at the ordinary mass is not affected by the existence of the intermediate hierarchies of interactions. Such a situation will remain to be the case for various kinds of grand unified theories.

Although Dawson and Georgi focused their attention on estimating the unification mass M independently of the details of the gauge hierarchy, it is also very important to find out relations between α, α_a and $\sin^2\theta_W$ renormalized at the ordinary mass M_W, which are, intimately but in a simple form, connected with possible structures of interaction hierarchies. These relations would provide a useful information how one could enlarge the gauge groups consistently with the 'known' group $SU(3)_C \times SU(2)_W \times U(1)$. In fact, as pointed out by Buras, Ellis, Gaillard and Nanopoulos (BEGN),[33] if one tries to enlarge the weak gauge group from SU(2) to a bigger group like SU(3), one would have to enlarge the strong colour group from SU(3) to SU(4) for example. Such a situation can be easily understood in the following way: If the weak SU(2) group is enlarged over a some typical mass scale, the corresponding effective coupling constant $g_2(\mu)$ will decrease faster with increasing μ than before. Since the unification mass M must be larger than several times 10^{14}GeV independently of details of interactions below M, the strong colour SU(3) group will be also obliged to increase so that the corresponding coupling constant $g_3(\mu)$ decreases faster and can coincide with $g_2(\mu)$ at the unification mass M. Taking a 'from the ground up' approach (enlarging the 'known' group and seeing what emerges) and tracing the trajectories of the effective coupling constants associated with the individual subgroups, Goldman and Ross (GR)[34] have investigated testable alternatives to SU(5) and found interesting results for the possible mass scales of new interactions in the intermediate region.

It is our purpose of this section to investigate systematically possible structures of the gauge hierarchy from such viewpoint as a 'from ground up' approach by tracing the trajectories of the effective coupling constants. In our analysis of the trajectories $g_i(\mu)$, the following two conditions should be kept in mind:

(i) At the starting point $\mu=M_W$, the coupling constants $g_i(\mu)$ should be consistent with the 'experimental' α, α_s and $\sin^2\theta_W$.

(ii) At the unification point $\mu=M$, the $g_i(\mu)$ should coincide with each other, i.e., $g_1=g_2=g_3$.

In §2.2, we recapitulate briefly DG results for completeness and reform them suitably for our purpose. Then applying it to the SU(N) model of grand unification, we derive another sum rule which could show clearly and quantitatively the correlation between the enlargement of the colour group and that of the flavour one, in addition to the relation obtained by DG. Furthermore, we obtain also similar sum rules for a modified SU(N) model with $\sin^2\theta_W(M)\neq 3/8$. Using these sum rules, we analyze numerically possible structures of the interaction hierarchy for the case in which there appears one or two gauge threshold in the intermediate region (§2.3). The last subsection (§2.4) is devoted to summary and discussion.

§2.2 Sum rules for the hierarchy of interactions

In this subsection, we first recapitulate briefly the formalism for the general hierarchy of interactions and write down the expression for trajectories of the effective coupling constants following Dawson and Georgi. Secondly, as an illustration, we adopt the SU(N) model of grand unification and derive sum rules restricting possible structures of the gauge hierarchy by making use of suitable linear combinations of the coupling constant trajectories.

Suppose that the 'known' $SU(3)_C\times SU(2)_W\times U(1)$ is enlarged step by step and unified to the grand unification group G in L steps, and that the gauge hierarchy is characterized by L+1 masses, μ_x $(x=0,1,2,\cdots,L;\mu_{x-1}\ll\mu_x)$ in such a way that the physics in a region between μ_{x-1} and μ_x can be described by an effective field theory[25] with an S^x (a subgroup of G) gauge symmetry. This situation can be shown schematically as follows:

$$S^0 \underset{\mu_0}{|} S^1 \underset{\mu_1}{|} S^2 \underset{\mu_2}{|} \cdots \underset{\mu_{x-1}}{|} S^x \underset{\mu_x}{|} \cdots \underset{\mu_{L-1}}{|} S^L \underset{\mu_L}{|} G ,$$

where $\mu_0=M_W$, $\mu_L=M$, $S^0=SU(3)_C\times U(1)_{em}$ and $S^1=SU(3)_C\times SU(2)_W\times U(1)$. The subgroup S^x is a product of simple non-Abelian subgroups or U(1)'s:

$$S^x = \prod_\alpha s^x_\alpha \quad , \tag{2·1}$$

where s^x_α is either a simple non-Abelian subgroup or a U(1), and we can take, for example, $s^0_1=U(1)_{em}$, $s^0_3=SU(3)_C$, $s^1_1=U(1)$,

$s_2^1=SU(2)_W$ and $s_3^1=SU(3)_C$.

The i-th generator $T^x_{\alpha i}$, corresponding to any convenient representation of G, of the subgroup s^x_α is normalized by

$$\mathrm{Tr}(\ T^x_{\alpha i}\ T^x_{\beta j}\ ) = \lambda \delta_{\alpha\beta}\delta_{ij} \tag{2·2}$$

with any convenient constant λ, and can be expressed by the linear combination of some subset of the generators $T^y_{\beta j}$ of S^y for y>x as follows:

$$T^x_{\alpha i} = \sum_{\beta,j} C^{x,y}_{\alpha i\beta j}\ T^y_{\beta j}\ . \tag{2·3}$$

Since only one of U(1) factors is available for the electromagnetic $U(1)_{em}$, we may assume that S^x contains at most one U(1) subgroup. From the gauge invariance in addition to the above restriction for each S^x, the coefficients $C^{x,y}_{\alpha i\beta j}$ satisfy the following relation,

$$\sum_k C^{x,y}_{\alpha i\gamma k}\ C^{x,y}_{\beta j\gamma k} = \delta_{\alpha\beta}\delta_{ij}P^{x,y}_{\alpha\gamma}\ , \tag{2·4}$$

where

$$P^{x,y}_{\alpha\beta} \equiv \sum_j \mid C^{x,y}_{\alpha i\beta j} \mid^2\ . \tag{2·5}$$

$P^{x,y}_{\alpha\beta}$ is the probability that the s^x_α subgroup of S^x exists in the s^y_β subgroup of S^y. From Eqs.(2·2), (2·3) and (2·5), the

$P^{x,y}_{\alpha\beta}$'s satisfy

$$\left.\begin{aligned} P^{x,x}_{\alpha\beta} &= \delta_{\alpha\beta}, \qquad \sum_{\beta} P^{x,y}_{\alpha\beta} = 1, \\ P^{x,y}_{\alpha\beta} &= \sum_{\gamma} P^{x,z}_{\alpha\gamma} P^{z,y}_{\gamma\beta}, \quad x \leq z \leq y . \end{aligned}\right\} \tag{2·6}$$

By taking account of Eqs.(2·4) and (2·5), Eq.(2·3) can be rewritten in a relation between the effective coupling constants renormalized at $\mu \geq \mu_{y-1}$, $g^x_\alpha(\mu)$ of s^x_α and $g^y_\beta(\mu)$ of s^y_β,

$$g^x_\alpha(\mu)^{-2} = \sum_{\beta} P^{x,y}_{\alpha\beta} \cdot g^y_\beta(\mu)^{-2} . \quad (\mu \geq \mu_{y-1}, \; y > x) \tag{2·7}$$

On the other hand, the renormalization equation for the $g^z_\alpha(\mu)$ gives

$$g^z_\alpha(\mu_{z-1})^{-2} = g^z_\alpha(\mu_z)^{-2} + 2b^z_\alpha \ln(\mu_z/\mu_{z-1}) \; , \tag{2·8}$$

where b^z_α is the constant which appears in the β-function for g^z_α,

$$\beta^z_\alpha(g^z_\alpha) = b^z_\alpha (g^z_\alpha)^3 + O((g^z_\alpha)^5) \; . \tag{2·9}$$

By combining Eqs.(2·7) and (2·8), the gauge coupling constants $g^x_\alpha(\mu)$ $(\mu_{x-1} \leq \mu \leq \mu_x)$ can be expressed in connection with parameters in different regions as follows:

$$g^x_\alpha(\mu_{x-1})^{-2} = g(\mu_L)^{-2} + \sum_{y=x} \ln(\mu_y/\mu_{y-1}) \; P^{x,y}_{\alpha\beta} 2b^y_\beta \; , \tag{2·10}$$

where g is the gauge coupling constant of the unifying group G. This is just the result obtained by DG and useful for analysing possible structures of the gauge hierarchy.

For the later application, we rewrite Eq.(2·10) in the following form:

$$g_\alpha^x(\mu_{x-1})^{-2} = g(\mu_L)^{-2} + (1/4\pi)\{K_\alpha^{x,x}t_{x-1} + \sum_{y=x}^{L-1} L_\alpha^{x,y}t_y\} \ , \qquad (2\cdot 11)$$

where

$$\left.\begin{array}{l} K_\alpha^{x,y}/4\pi = \sum_\beta P_{\alpha\beta}^{x,y}\cdot 2b_\beta^y, \quad L_\alpha^{x,y} = K_\alpha^{x,y+1} - K_\alpha^{x,y}, \\ t = \ln(\mu_L/\mu), \quad t_y = \ln(\mu_L/\mu_y). \quad (\ y \geq x\ ) \end{array}\right\} \qquad (2\cdot 12)$$

In order to obtain some relations among relevant quantities, α, α_s and $\sin^2\theta_W$, we first write down Eq.(2·11) for x=1:

$$4\pi g_1^1(\mu_0)^{-2} = 4\pi g(\mu_L)^{-2} + K_1^{1,1}t_0 + \sum_{y=1}^{L-1} L_1^{1,y}t_y \ , \qquad (2\cdot 13\cdot a)$$

$$4\pi g_2^1(\mu_0)^{-2} = 4\pi g(\mu_L)^{-2} + K_2^{1,1}t_0 + \sum_{y=1}^{L-1} L_2^{1,y}t_y \ , \qquad (2\cdot 13\cdot b)$$

$$4\pi g_3^1(\mu_0)^{-2} = 4\pi g(\mu_L)^{-2} + K_3^{1,1}t_0 + \sum_{y=1}^{L-1} L_3^{1,y}t_y \ , \qquad (2\cdot 13\cdot c)$$

which can be expressed in terms of α, α_s and $\sin^2\theta_W$ if one uses the following usual relations:

$$\left.\begin{aligned} g_1^1(\mu_0)^2/4\pi &= C^2\cdot\alpha(\mu_0)/\cos^2\theta_W \ , \\ g_2^1(\mu_0)^2/4\pi &= \alpha(\mu_0)/\sin^2\theta_W \ , \\ g_3^1(\mu_0)^2/4\pi &= \alpha_s(\mu_0) \ . \end{aligned}\right\} \qquad (2\cdot14)$$

Here the factor C fixes the normalization of the weak hypercharge Y together with the third component I_3 of the weak isospin in connection with the electric charge operator Q,

$$Q = I_3 + Y/2 \equiv T_{23}^1 - C\, T_{11}^1 \ , \qquad (2\cdot15)$$

where T_{23}^1 and T_{11}^1 are the generators of the $SU(2)_W$ and the U(1) respectively, and $C=-\sqrt{5/3}$ for the SU(5). Next, eliminating $4\pi g(\mu_L)^{-2}$ by the use of suitable linear combinations of Eqs.(2·13·a)∿(2·13·c), we can obtain two independent sum rules which connect the observed α, α_s and $\sin^2\theta_W$ with possible structures of the gauge hierarchy. To obtain concrete forms of sum rules and clarify the implication of them, we investigate in detail two cases, the SU(N) model of DG and its modified version in what follows.

(I) *The SU(N) model*

Dawson and Georgi considered the following SU(N) model:

$$\left.\begin{array}{l} s_1^x = U(1)^x, \quad s_2^x = SU(m_x) \supset SU(2)_W \ , \\ s_3^x = SU(n_x) \supset SU(3)_C \ , \quad G = SU(N) \supset SU(5) \ , \end{array}\right\} \quad (2\cdot16)$$

where the N-dimensional representation of G consists of an SU(5) **5** and N-5 neutral singlets and therefore $C=-\sqrt{5/3}$. The relevant $P^{x,y}_{\alpha\beta}$ for the SU(N) model is easily calculated by using N×N matrices $T^x_{\alpha i}$ for the fundamental representation of SU(N). For example, the generators T^1_{11}, T^1_{23} and the related charge operator Q are expressed by the following diagonal N×N matrices with the normalization constant $\lambda=1/2$ (see Eq.(2·2)):

$$\left.\begin{array}{l} \sqrt{5/3}\ T^1_{11}=\text{diag.}(0,\cdots 0,-1/3,-1/3,-1/3,\ 1/2,\ 1/2,0,\cdots 0), \\ T^1_{23}=\text{diag.}(0,\cdots 0,\ 0\ ,\ 0\ ,\ 0\ ,\ 1/2,-1/2,0,\cdots 0), \\ Q=\text{diag.}(0,\cdots 0,-1/3,-1/3,-1/3,\ 1\ ,\ 0\ ,0,\cdots 0). \end{array}\right\} \quad (2\cdot17)$$

The calculated results for $P^{1,x}_{\alpha\beta}$ are

$$\left.\begin{array}{ll} P^{1,x}_{22}=P^{1,x}_{33}=1\ , & P^{1,x}_{11}=(6/5)(m_x+n_x)/m_x n_x\ , \\ P^{1,x}_{12}=(3/5)(m_x-2)/m_x\ , & P^{1,x}_{13}=(2/5)(n_x-3)/n_x\ . \quad (x\geq 1) \end{array}\right\} \quad (2\cdot18)$$

As is well known,[18] the coefficient b^y_β required for $K^{x,y}_\alpha$ and

$L_\alpha^{x,y}$ are given,

$$\left.\begin{aligned} &48\pi^2 b_1^y = 2F_1^y\ , \qquad 48\pi^2 b_2^y = -11m_y + 2F_2^y\ , \\ &48\pi^2 b_3^y = -11n_y + 2F_3^y\ , \end{aligned}\right\} \tag{2·19}$$

where the constants F_α^y depend on the number of spin 1/2 and scalar particles with mass less than μ_y, and are assumed to be independent of α ($F_1^y=F_2^y=F_3^y$) following DG. Combining Eqs.(2·12), (2·18) and (2·19), we obtain $K_\alpha^{1,x}$ and $L_\alpha^{1,x}$ as follows:

$$\left.\begin{aligned} &K_1^{1,x}=-(11/30\pi)\{3(m_x-2)+2(n_x-3)\}+8\pi b_1^x, \\ &K_2^{1,x}=-(11/6\pi)m_x+8\pi b_1^x, \quad K_3^{1,x}=-(11/6\pi)n_x+8\pi b_1^x, \quad (x\geq 1) \end{aligned}\right\} \tag{2·20}$$

$$\left.\begin{aligned} &L_1^{1,x}=-(11/30\pi)\{3\Delta m_x+2\Delta n_x\}+8\pi\Delta b_1^x, \\ &L_2^{1,x}=-(11/6\pi)\Delta m_x+8\pi\Delta b_1^x, \quad L_3^{1,x}=-(11/6\pi)\Delta n_x+8\pi\Delta b_1^x, \quad (x\geq 1) \end{aligned}\right\} \tag{2·21}$$

where

$$\Delta m_x=m_{x+1}-m_x, \quad \Delta n_x=n_{x+1}-n_x, \quad \Delta b_1^x=b_1^{x+1}-b_1^x. \tag{2·22}$$

If one notices two relations, $(5/3)K_1^{1,1}-K_2^{1,1}-(2/3)K_3^{1,1}=(22/3\pi)$ and $(5/3)L_1^{1,y}-L_2^{1,y}-(2/3)L_3^{1,y}=0$, which follow from Eqs.(2·20) and (2·21) respectively, one obtains by a combination

(5/3)(2·13·a)−(2·13·b)−(2/3)(2·13·c) the following relation:

$$\cos 2\theta_W/\alpha(M_W) - (2/3)/\alpha_s(M_W) = (22/3\pi)\ln(M/M_W) \ , \qquad (2\cdot 23)$$

which is nothing but the DG relation. On the other hand, if one uses another combination 3(2·13·b)−(2·13·a)−2(2·13·c) by taking account of a relation $3K_2^{1,1} - K_1^{1,1} - 2K_3^{1,1} = 0$, one can eliminate $4\pi g(\mu_L)^{-2}$ and t_0-terms simultaneously from Eqs.(2·13·a) ∿(2·13·c), and obtains another sum rule

$$\sum_{x=1}^{L-1} (\Delta_{x+1} - \Delta_x) t_x = (3\pi/22)\{(6\sin^2\theta_W - 1)/\alpha(M_W) - (10/3)/\alpha_s(M_W)\} \equiv \kappa_1 , \qquad (2\cdot 24)$$

where $\Delta_x = n_x - m_x$ is the difference of 'colour' from 'flavour' in each step of effective interactions. The right-hand side of this sum rule, κ_1, is a function of 'known' quantities $\alpha(M_W)$, $\alpha_s(M_W)$, $\sin^2\theta_W$ and is rather small, while the left-hand side of it shows that the enlargement of the colour and flavour groups in each step is closely related to the mass scale μ_x to give a small $|\kappa_1|$ as a whole. These circumstances of the grand unification have been pointed out qualitatively by BEGN.[33)]

(II) *The modified SU(N) model*

Various grand unified theories have predicted $\sin^2\theta_W = 3/8$ ($C = -\sqrt{5/3}$) at the unification mass M, and Dawson and Georgi also have considered the SU(N) model with $C = -\sqrt{5/3}$ in which the N-dimensional representation consists of an SU(5) **5** and N-5 neutral singlets. However, there is no a priori reason why it should be the case. Therefore, it would be very interesting to investigate if the model with $C \neq -\sqrt{5/3}$ could be constructed consistently with two conditions for the coupling constant trajectories mentioned in Introduction of this section (§2.1).

Among various possibilities we take here as a simplest illustration a modified SU(N) model, the fundamental representation of which is characterized by the following diagonal N×N matrices:

$$\left.\begin{aligned}
\sqrt{(5+12q^2)/3}\ T^1_{11} &= \text{diag.}(0,\cdots q,-1/3,-1/3,-1/3,\ 1/2,\ 1/2,-q,0,\cdots 0),\\
T^1_{23} &= \text{diag.}(0,\cdots 0,\ \ 0\ ,\ \ 0\ ,\ \ 0\ ,\ 1/2,-1/2,\ 0,0,\cdots 0),\\
Q &= \text{diag.}(0,\cdots q,-1/3,-1/3,-1/3,\ \ 1\ ,\ \ 0\ ,-q,0,\cdots 0).
\end{aligned}\right\} \tag{2·25}$$

This model is just the DG SU(N) model except that two of the N-5 neutral singlets in the latter are replaced with charged ones of the charge $\pm q$ in the former, and gives $\sin^2\theta_W = 3/(8+12q^2) \neq 3/8$ $(C=-\sqrt{(5+12q^2)/3}\ )$ at the unification mass M. Furthermore in what follows, we assume for simplicity that both the flavour and colour groups are enlarged simultaneously at μ_1 in such a way as $s^2_2 \supset SU(3)$, $s^2_3 \supset SU(4)$. In order to obtain sum rules for this model, we must calculate $P^{1,x}_{\alpha\beta}$, $K^{1,x}_{\alpha}$ and $L^{1,x}_{\alpha}$. The expressions of $P^{1,x}_{\alpha\beta}$, $K^{1,x}_{\alpha}$, $L^{1,x}_{\alpha}$ for $\alpha=2,3$, $P^{1,1}_{1\alpha}$ and $K^{1,1}_{1}$ are the same as those of the DG model (see Eqs.(2·18), (2·20) and (2·21)). The results different from those of the DG model are as follows:

$$\left.\begin{aligned}
P_{11}^{1,x} &= 2(1-q)^2 C^{-2}(m_x+n_x)/m_x n_x \ , \\
P_{12}^{1,x} &= C^{-2}\{(1+2q^2)m_x-2(1-q)^2\}/m_x \ , \\
P_{13}^{1,x} &= (2/3)C^{-2}\{(1+3q^2)n_x-3(1-q)^2\}/n_x \ , \quad (x \geq 2)
\end{aligned}\right\} \quad (2\cdot26)$$

$$K_1^{1,x} = -(11/6\pi)C^{-2}\{(1+2q^2)m_x+(2/3)(1+3q^2)n_x-4(1-q)^2\} + 8\pi b_1^x \ , \quad (x \geq 2) \quad (2\cdot27)$$

$$\left.\begin{aligned}
L_1^{1,1} &= -(11/6\pi)C^{-2}\{(1+2q^2)\Delta m_1+(2/3)(1+3q^2)\Delta n_1+2q(4+3q)\} \\
&\quad + 8\pi\Delta b_1^1 \ , \\
L_1^{1,x} &= -(11/6\pi)C^{-2}\{(1+2q^2)\Delta m_x+(2/3)(1+3q^2)\Delta n_x\} \\
&\quad + 8\pi\Delta b_1^x \ . \quad (x \geq 2)
\end{aligned}\right\} \quad (2\cdot28)$$

Using these results and making use of linear combinations of Eqs.(2·13·a)$\sim$(2·13·c), $(5+12q^2)/3\cdot$(2·13·a)$-(1+2q^2)\cdot$(2·13·b) $-2(1+3q^2)/3\cdot$(2·13·c) and $3\cdot$(2·13·b)$-$(2·13·a)$-2\cdot$(2·13·c), we obtain the following two sum rules:

$$(1+5q^2/2)t_0-q(2+3q/2)t_1$$

$$=(3\pi/22)[\{1-2(1+q^2)\sin^2\theta_W\}/\alpha-2(1+3q^2)/3\alpha_s]\equiv\kappa_2(q), \qquad (2\cdot29)$$

$$q(2+3q/2)t_1+(1+5q^2/2)\sum_{x=1}^{L-1}(\Delta_{x+1}-\Delta_x)t_x$$

$$+(3\pi/22)[\{6(1+2q^2)\sin^2\theta_W-1\}/\alpha-2(5+12q^2)/3\alpha_s]\equiv\kappa_3(q). \qquad (2\cdot30)$$

The former of which is reduced to Eq.(2·23) and the latter to Eq.(2·24) when q=0. Addition of both sides of Eqs.(2·29) and (2·30) gives also another sum rule independent of q,

$$t_0+\sum_{x=1}^{L-1}(\Delta_{x+1}-\Delta_x)t_x=(6\pi/11)(\sin^2\theta_W/\alpha-1/\alpha_s)\equiv\kappa_4 . \qquad (2\cdot31)^{*)}$$

It should be remarked here that the left-hand side of Eq.(2·29) involves t_1 in addition to t_0 and therefore Eq.(2·29) unlike Eq.(2·23) is of no use directly for the estimation of the unification mass M. For special cases $\Delta_{x+1}-\Delta_x=0$, Eq.(2·31) is rather useful to estimate the mass M. In general, structures of the gauge hierarchy can be investigated by making use of two combinations of Eqs.(2·29), (2·30) and (2·31), and the

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*) Equation (2·31) can be derived by a combination Eq.(2·13·b) - Eq.(2·13·c), which is independent of $g_1(\mu)$ trajectory. This is the case for both ordinary and modified SU(N) models.

sum $\sum_{x=1}^{L-1} (\Delta_{x+1}-\Delta_x)t_x$ in these equations suggests the correlation between the enlargement of the colour degrees of freedom and those of the flavour.

The model considered above introduces *new* Weyl spinors with the electric charge q, q-1/3 etc. in addition to those of the SU(5)[21] of Georgi and Glashow. If this model should be realistic, these new particles also should have their chiral partners and therefore all the charged particles should be massive from the requirement of no anomalies. In fact, for example, a technicoloured grand unification model (G=SU(7) with s_2^2=SU(2) and s_3^2=SU(5) rather than our s_2^2=SU(3) and s_3^2=SU(4)) by Farhi and Susskind,[35] takes a choice [2]+[4]+[6] as an anomaly free combination, where [m] denotes the totally anti-symmetric representation of rank m and the only massless particle is the ν_e. Here, we do not enter into details of the model, which will be discussed in §§3 and 4.

§2.3 Numerical analysis for the possible steps (μ_x) characterizing the gauge hierarchy

In this subsection, we investigate in detail structures of the gauge hierarchy for the case where there is one step or two between μ_0 and M by using sum rules obtained in the preceding subsection (§2.2). For the numerical analysis, we use the following 'experimental' values[14),26),27)] as the starting values at $\mu_0=M_W\simeq100$GeV of the coupling constant trajectories:

$$\alpha = 1/128, \quad \alpha_s = 0.08 \sim 0.18, \quad \sin^2\theta_W = 0.23 \pm 0.02, \tag{2·32}$$

which are renormalized at $\mu=M_W$.

(I) *The SU(N) model*

(i) One step between M_W and M $(L=2)$

For $L=2$, Eq.(2·24) turns out to be

$$(\Delta_2-\Delta_1)t_1 = \kappa_1(\alpha,\alpha_s,\sin^2\theta_W) \ , \tag{2·33}$$

where $\Delta_1=n_1-m_1=1$, $\Delta_2=n_2-m_2=$integer.

For $\Delta_1=\Delta_2$ (or furthermore for $\Delta_{x+1}=\Delta_x$), $\kappa_1(\alpha,\alpha_s,\sin^2\theta_W)=0$ together with Eq.(2·23) is reduced to the original result by GQW, and the constraint for μ_1 (or for $\mu_x;x=1,2,\cdots L-1$) disappears. The result, $\kappa_1(\alpha,\alpha_s,\sin^2\theta_W)=0$, however, seems to be too accidental.

For $\Delta_2\neq\Delta_1$, the sign of $(\Delta_2-\Delta_1)$ is the same as that of κ_1 because of $t_1>0$ $(M>>\mu_1)$. In Fig.2, we plot the curve $\kappa_1(\alpha,\alpha_s,\sin^2\theta_W)=0$ and the proton lifetime $\tau_p=\tau_p(M)=\tau_p(\mu_0,\alpha,\alpha_s,\sin^2\theta_W)\simeq10^{30}$ years $(M=3\times10^{14}\text{GeV})$[23),26)] together with allowed 'experimental' region (Eq.(2·32)) of α_s and $\sin^2\theta_W$ in the α_s-$\sin^2\theta_W$ plane. As is easily seen from Fig.2, the sign of κ_1 consistent with the proton lifetime (or M) and the allowed

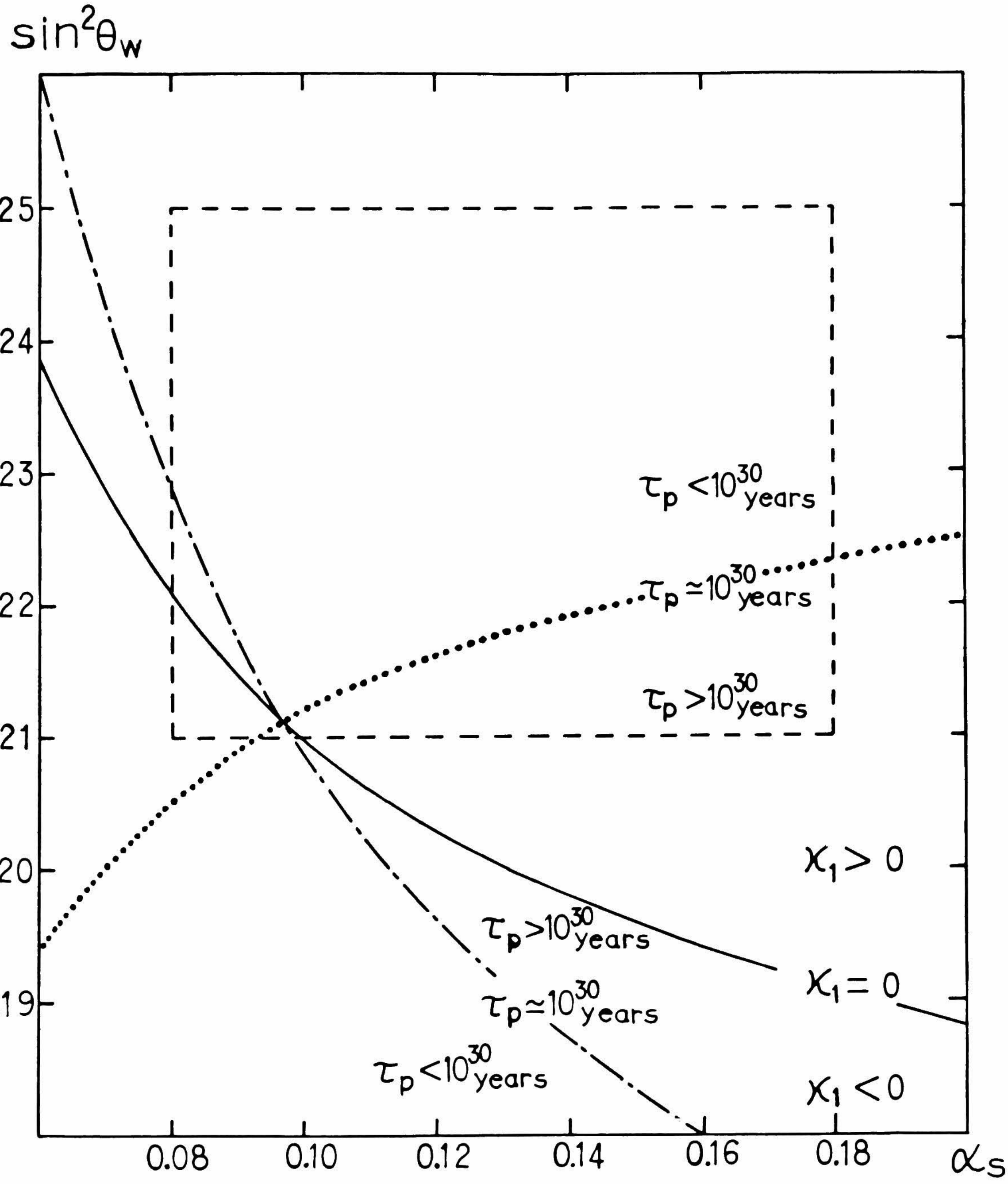

Fig.2. Proton lifetime τ_p and κ_1 as functions of α_s and $\sin^2\theta_W$. The dotted curve, $\tau_p=10^{30}$years, is drawn by using Eq.(2·23), while the dash dotted by Eq.(2·37). The region surrounded by dashed lines represents $\sin^2\theta_W=0.23\pm0.02$ and $\alpha_s=0.08\sim0.18$.

region of $(\alpha_s, \sin^2\theta_W)$ is positive on the whole, and then the magnitude of $\sin^2\theta_W$ is rather small, $\sin^2\theta_W \lesssim 0.22$, which is independent of L.

In Fig.3, for given values (1 and 2) of $|\Delta_2-1|$, we plot M-$\mu_1(M_1)$ relations corresponding to the fixed $\sin^2\theta_W$(changing α_s) or α_s (changing $\sin^2\theta_W$), and tabulate the numerical values of the typical cases in Table I. From Fig.3, we can see that for $|\Delta_2-1|=2$, M_1 is not so far from M compatible with the proton lifetime and for $|\Delta_2-1|\geq 3$, the condition $\mu_1 \ll M$ can no longer be satisfied. Therefore, in one step case, the difference of 'colour' from 'flavour' $\Delta_2=n_2-m_2$ is at most 3, suggesting the correlation between enlargement of colour and that of flavour. Even in the most favourable case $|\Delta_2-1|=1$, for example, $\Delta_2=2$, the range of M_1 consistent with $\tau_p \gtrsim 10^{30}$ years and conditions (2·32) for α, α_s, $\sin^2\theta_W$ is $M_1 \gtrsim 10^{10}$ GeV, which would be out of the feasible test of new interactions.

(ii) Two steps between M_W and M (L=3)

For L=3, Eq.(2·24) becomes

$$(\Delta_2-\Delta_1)t_1 + (\Delta_3-\Delta_2)t_2 = \kappa_1(\alpha,\alpha_s,\sin^2\theta_W) , \qquad (2\cdot 34)$$

where

$$\left.\begin{array}{l} t_0=\ln(M/M_W),\ t_1=\ln(M/M_1),\ t_2=\ln(M/M_2), \\ 0 < t_2 < t_1 < t_0 ,\quad \text{or}\quad M_W \ll M_1 \ll M_2 \ll M . \end{array}\right\} \qquad (2\cdot 35)$$

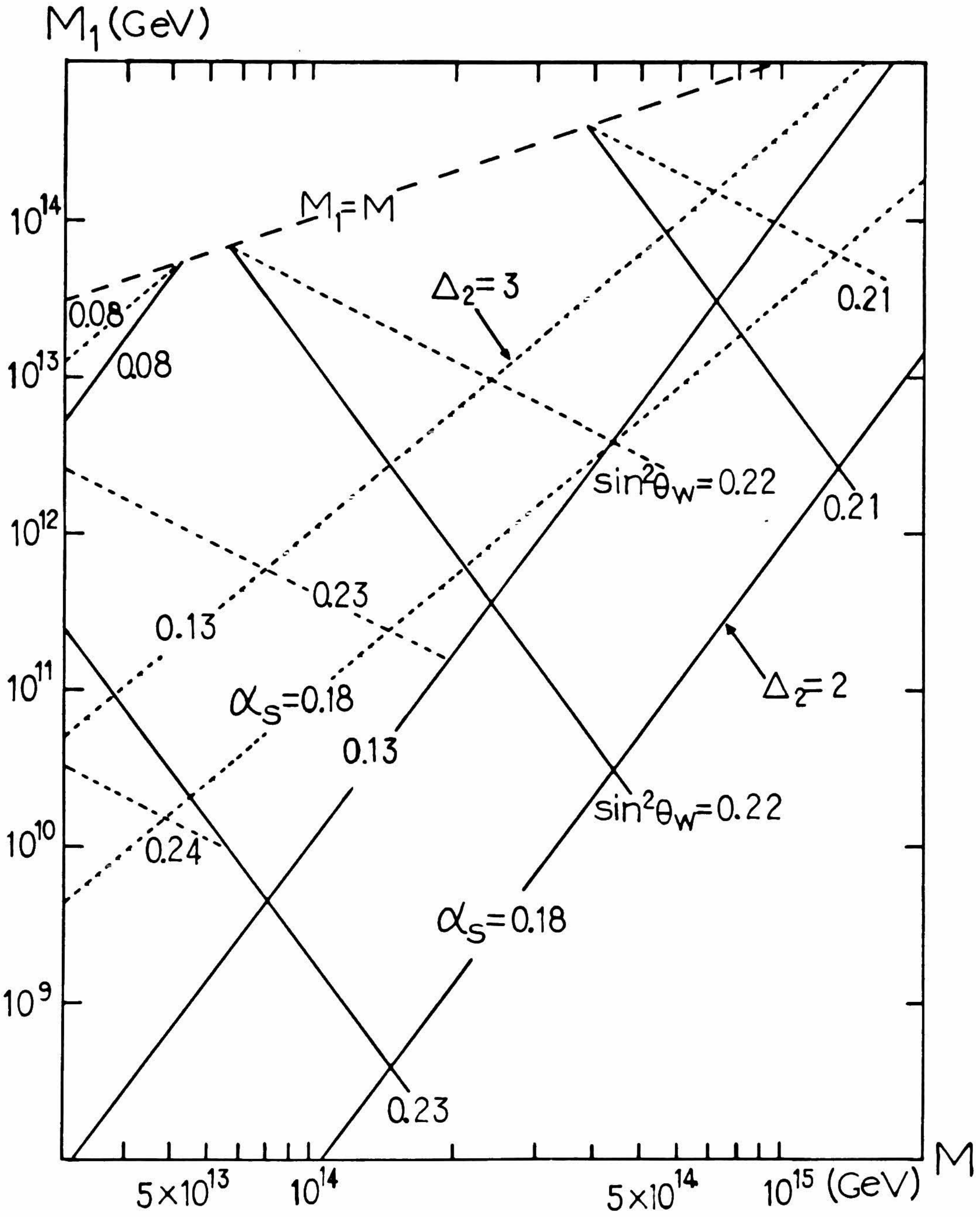

Fig.3. M-M_1 relation for the one-step enlargement to the SU(N). A set of solid lines are written for Δ_2=2 and that of dotted ones for Δ_2=3, by fixing $\sin^2\theta_W$(changing α_s) or α_s(changing $\sin^2\theta_W$). The dashed line represents the boundary M $\gtrless$ M_1.

Table I. Typical values of κ_1, M and M_1 for the one-step enlargement to the SU(N). For negative κ_1, Δ_2=2 and Δ_2=3 should be read as Δ_2=0 and Δ_2=-1, respectively.

$\sin^2\theta_W$	α_s	κ_1	M (GeV)	M_1 (GeV)	
				$\Delta_2 = 2$	$\Delta_2 = 3$
0.21	0.08	-3.6	1.8×10^{14}	5.0×10^{12}	3.0×10^{13}
	0.13	3.3	7.2×10^{14}	2.7×10^{13}	1.4×10^{14}
	0.18	6.3	1.3×10^{15}	2.4×10^{12}	6.5×10^{13}
0.22	0.08	-0.3	6.1×10^{13}	4.5×10^{13}	5.2×10^{13}
	0.13	6.6	2.4×10^{14}	3.4×10^{11}	9.1×10^{12}
	0.18	9.6	4.4×10^{14}	3.0×10^{10}	3.6×10^{12}

For respective α_s and $\sin^2\theta_W$ (α fixed), possible t_1 and t_2 are determined by Eqs.(2·34) and (2·35) together with t_0 estimated by Eq.(2·23). For a given t_0, t_1 and t_2 satisfying Eq.(2·35) lie in the inner region of the triangle surrounded by three lines, $t_1=t_0$, $t_1=t_2$, $t_2=0$, in the t_1-t_2 plane. At the same time, the t_1 and t_2 should satisfy Eq.(2·34), i.e., the line (2·34) with $\kappa_1>0$ *) should pass through the inner region of the triangle. From the above geometrical consideration of Eqs.(2·34) and (2·35), possible solutions occur only when a pair of signs [sign($\Delta_2-\Delta_1$),sign($\Delta_3-\Delta_2$)] are following: [0,+], [+,0], [+,+], [+,-], [-,+].

In the case [0,+], there is no constraint for $M_1(t_1)$ except for Eq.(2·35), and $t_2(M_2)$ should satisfy the condition $(\Delta_3-\Delta_2)t_2=\kappa_1$, which is the same as that of one step case (L=2). Possible solutions for t_2 can be obtained for $2\geq(\Delta_3-\Delta_2)\geq1$. For $\Delta_3-\Delta_2=2$, however, M_2 is not far from M, as is easily recognized from the analysis of one step case. Thus, the case [0,+] is reduced to the case $\Delta_2-\Delta_1=0$, $\Delta_3-\Delta_2=1$, and a reasonable situation $M_W<<M_1<<M_2<<M$ (of course, $M_W<M_1<<M_2<<M$ also **)) is realized although a situation $M_W<<M_1<<M_2<M$ is possible even for $\Delta_2-\Delta_1=0$, $\Delta_3-\Delta_2=2$. By the same reasoning as that of the case [0,+], the case [+,0] is reduced to the one $2\geq(\Delta_2-\Delta_1)\geq1$,

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*) As is stated above, the sign of κ_1 is positive on the whole.

**) $M_W<M_1$ implies that there is a chance for the feasible test of new interactions.

where $10^{10}\text{GeV} \lesssim M_1 << M$ $(M_1 < M)$ for $\Delta_2 - \Delta_1 = 1$ $(\Delta_2 - \Delta_1 = 2)$ and M_2 is arbitrary. This implies $M_W << M_1 < M_2 << M$ or $M_W << M_1 << M_2 < M$, and therefore the case [+,0] is unacceptable. As for the case [+,+], $\Delta_2 - \Delta_1 \geq 1$, $\Delta_3 - \Delta_2 \geq 1$ and $t_1 > t_2$, Eq.(2·34) can be rewritten in the following inequality: $t_2 < \kappa_1 / \{(\Delta_2 - \Delta_1) + (\Delta_3 - \Delta_2)\} \leq \kappa_1 / 2$. This inequality implies $M \times 10^{-2} < M_2$ since $\kappa_1 / 2 \lesssim 5$ for reasonable variation of α_s and $\sin^2\theta_W$. Thus the case [+,+] is also excluded as a two-step case.

In the final case [-,+] ([+,-]), much colour (flavour) freedom is enlarged at M_2 to compensate the effects which are caused by the enlargement of much flavour (colour) at M_1. In order to investigate concretely such a case, we try to reproduce, as an example, GR analysis for technicolour theories by using our sum rule (2·34). The technicolour theories are characterized as follows: $SU(3)_C \times SU(2)_W \times U(1)$ $[\times SU(4)_{T.C.}] \rightarrow SU(3)_C \times SU(m_2)_W \times U(1)[\times SU(4)_{T.C.}]$ at M_1, $SU(3)_C \times SU(m_2)_W \times U(1)[\times SU(4)_{T.C.}] \rightarrow SU(7)_C \times SU(m_2)_W \times U(1)$ at M_2, where $m_2 \geq 3$, $\Delta_2 - \Delta_1 = 2 - m_2$, $\Delta_3 - \Delta_2 = 4$. Furthermore, GR estimated M_2 to be $10^9 \sim 10^{11}\text{GeV}$ from conditions, $\alpha_{T.C.}(1\text{TeV}) \sim O(1)$ and $\alpha_{T.C.}(M_2) = \alpha_s(M_2)$ $(\alpha_{T.C.} = g^2_{T.C.}/4\pi)$. We can estimate M_1 from Eq.(2·34) for various values of m_2. The results are tabulated in Table II. Since $M_1 \lesssim M_W$ for $m_2 = 3$ and $M_1 > M_2$ for $m_2 \geq 7$, the values m_2 giving reasonable M_1 are 4, 5 and 6, which is just the result obtained by GR. For $m_2 = 6$, however, M_1 is comparable with M_2 like one step case unless $M \gtrsim 10^{16}\text{GeV}$ corresponding

Table II. Typical values of M and M_1 for the technicolour model. Typical values are calculated for $\sin^2\theta_W=0.21$, $\alpha_s=0.10$, 0.15 and $M_2=10^9$GeV, 10^{11}GeV.

m_2	α_s	M (GeV)	M_1 (GeV)	
			$M_2=10^9$GeV	$M_2=10^{11}$GeV
3	0.10	3.7×10^{14}	1.9×10^{-8}	1.9
	0.15	9.7×10^{14}	1.3×10^{-7}	1.7×10
4	0.10	3.7×10^{14}	2.6×10^{3}	2.6×10^{7}
	0.15	9.7×10^{14}	1.1×10^{4}	1.1×10^{8}
5	0.10	3.7×10^{14}	1.4×10^{7}	6.4×10^{9}
	0.15	9.7×10^{14}	4.9×10^{7}	2.3×10^{10}
6	0.10	3.7×10^{14}	9.9×10^{8}	9.9×10^{10}
	0.15	9.7×10^{14}	3.3×10^{9}	3.3×10^{11}

to $\sin^2\theta_W \lesssim 0.19$. Here, we show, in Fig.4, the behaviours of the gauge coupling constants for the case of $m_2=4$, which corresponds to the breaking pattern

$$\begin{aligned} SU(11) &\to SU(7)_C \times SU(4)_W \times U(1) \text{ at } M \\ &\to SU(3)_C \times SU(4)_W \times U(1)\,[\times SU(4)_{T.C.}] \text{ at } M_2 \\ &\to SU(3)_C \times SU(2)_W \times U(1)\,[\times SU(4)_{T.C.}] \text{ at } M_1, \end{aligned}$$

as a comparison with those for the standard GUT SU(5) (Fig.1).

In order to understand unifiedly the above argument, let us investigate more systematically the enlargement of gauge groups conditioned by Eqs.(2·34) and (2·35). First, we rewrite condition (2·35) $0<t_2<t_1<t_0$ (or $M_W \ll M_1 \ll M_2 \ll M$) more quantitatively by replacing a relation $A \ll B$ with another $10^2 \cdot A \leq B$ as follows:

$$\left.\begin{array}{l} 10^2 \cdot M_W \leq M_1, \quad 10^2 \cdot M_1 \leq M_2, \quad 10^2 \cdot M_2 \leq M \\ \text{or} \\ t_1 \leq t_0 - \ln 10^2, \quad t_2 \leq t_1 - \ln 10^2, \quad 0 \leq t_2 - \ln 10^2, \end{array}\right\} \quad (2\cdot35)'$$

which represents a smaller triangle than the triangle (2·35) in the t_1-t_2 plane. Secondly varying (t_1, t_2) in the triangle (2·35)', we plot in Fig.5 the region allowed by Eq.(2·34) in the $(\Delta_2-\Delta_1)-(\Delta_3-\Delta_2)$ plane for $\alpha_s=0.15$ and $\sin^2\theta_W=0.22$. It should be remarked that the allowed region hardly changes for a reasonable variation of α_s and $\sin^2\theta_W$ since κ_1 is relatively

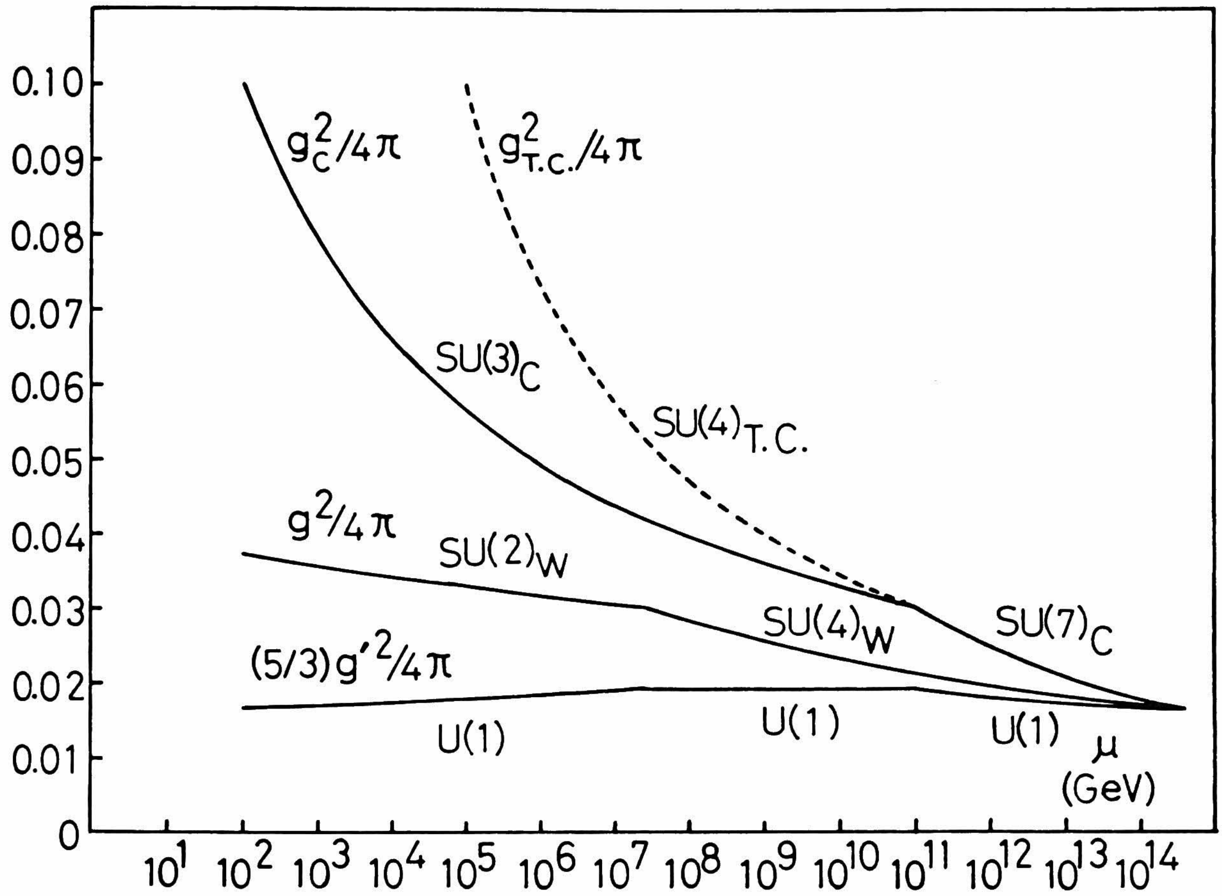

Fig.4. The energy dependences of gauge coupling constants in the SU(11) whose breaking pattern is SU(11) $\to$ SU(7)$_C$ $\times$SU(4)$_W$$\times$U(1) at M $\to$ SU(3)$_C$$\times$SU(4)$_W$$\times$U(1) [$\times$SU(4)$_{T.C.}$] at M_2 $\to$ SU(3)$_C$$\times$SU(2)$_W$$\times$U(1) [$\times$SU(4)$_{T.C.}$] at M_1.

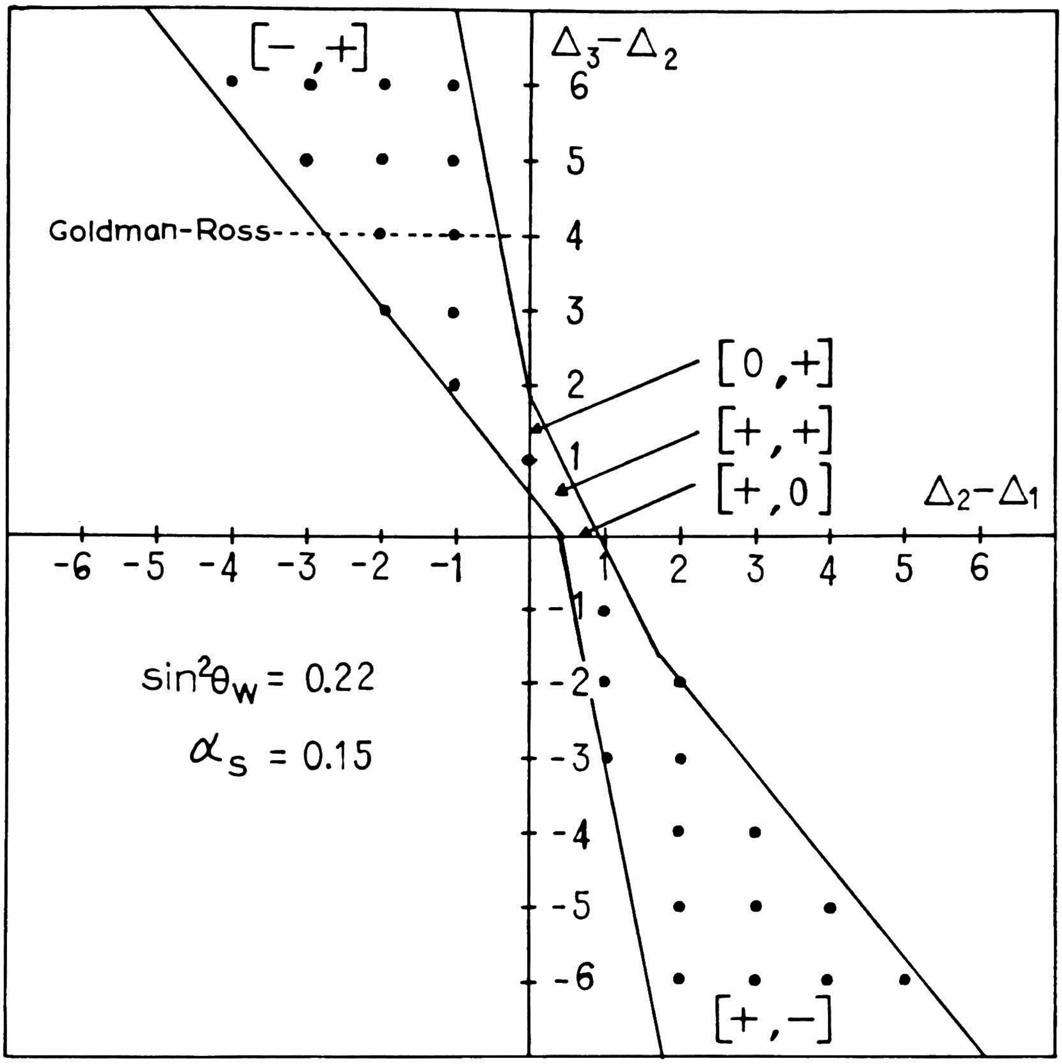

Fig.5. Two-step enlargement to the SU(N). Possible ways of the enlargement are represented by lattice points (•) in the region surrounded by lines.

small. Here, the lattice points in the allowed region correspond to possible ways for the enlargement of gauge groups.

As is easily seen from Fig.5 a type of enlargement $\Delta_2-\Delta_1=0$, $\Delta_3-\Delta_2=1$ is possible as the case [0,+1], but the cases [+,0] and [+,+] are excluded as possible enlargement, which is just our result mentioned above. The part $\Delta_3-\Delta_2=4$ in the second quadrant (Fig.5) corresponds to the technicolour theories analyzed by GR.[34] Possible solutions for $\Delta_3-\Delta_2=4$ are $\Delta_2-\Delta_1(=2-m_2)=-1,-2$, i.e., $m_2=3,4$. A solution $m_2=3$ is allowed since we do not use such a condition $M_2=10^9\sim10^{11}$GeV as GR did, while solutions $m_2=5,6$ are excluded by our analysis. The solutions $m_2=5,6$, however, may be allowed if the condition $M_1<<M_2$ is loosened in such a way as $M_1<M_2$. The solution $m_2=4$ lies nearly in the center of the allowed region with $\Delta_3-\Delta_2=4$, which suggests that M_1 can be relatively small in this case. As for the magnitude of $(\Delta_3-\Delta_2)$ and $(\Delta_2-\Delta_1)$ in cases [+,-] and [-,+], we can obtain no restriction from our analysis, but there is no reason to expect that the magnitude is very large.

At any rate, the above analysis shows that our sum rules give very useful information about possible structures of gauge hierarchy.

(II) *The modified SU(N) model*

We would like here to investigate how the result of model (I) is altered by modification of charge assignment for the fundamental representation. We take, for simplicity, one step case (L=2) of the following enlargement ($\Delta_2-\Delta_1=0$), $SU(3)_C\times SU(2)_W \times U(1) \rightarrow SU(4)_C\times SU(3)_W\times U(1)$ at M_1. Now, Eqs.(2·30) and (2·31) turn out to be

$$q(2+3q/2)t_1 = \kappa_3(q) \ , \tag{2·36}$$

$$t_0 = \kappa_4 \ . \tag{2·37}$$

It is very interesting to remark that the formula determing t_0 is the relation (2·37) instead of DG relation (2·23). This indicates the original DG relation will be considerably modified by $q\neq 0$. Noticing $\kappa_3(0)=\kappa_1$, it is easily shown that if q=0 and $\kappa_1=0$, Eqs.(2·36) and (2·37) (or course, $\Delta_2-\Delta_1=0$) can reproduce the one step case with $\Delta_2-\Delta_1=0$ of the model (I). The case $q\neq 0$, however, does give results considerably different from those of the model (I) even if $\Delta_2-\Delta_1=0$. That is, in the latter case, M_1 is arbitrary and M is determined by Eq.(2·23) as a function of α, α_s and $\sin^2\theta_W$, while in the former, M_1 is restricted by Eq.(2·36) and M is given by Eq.(2·37) different from Eq.(2·23) since $\kappa_1=0$ is not necessarily satisfied.

In Fig.6, M-M_1 relations are plotted, as an illustration, for q=1/3, 1/2 and 2/3 varying $\alpha_s=0.08\sim 0.18$ and $\sin^2\theta_W=0.21 \sim 0.25$, and some typical sets of values also are tabulated in Table III. The value of M_1 is relatively small, for example, $M_1=10^4\sim 10^9$GeV for q=1/3. For comparison with one step case of

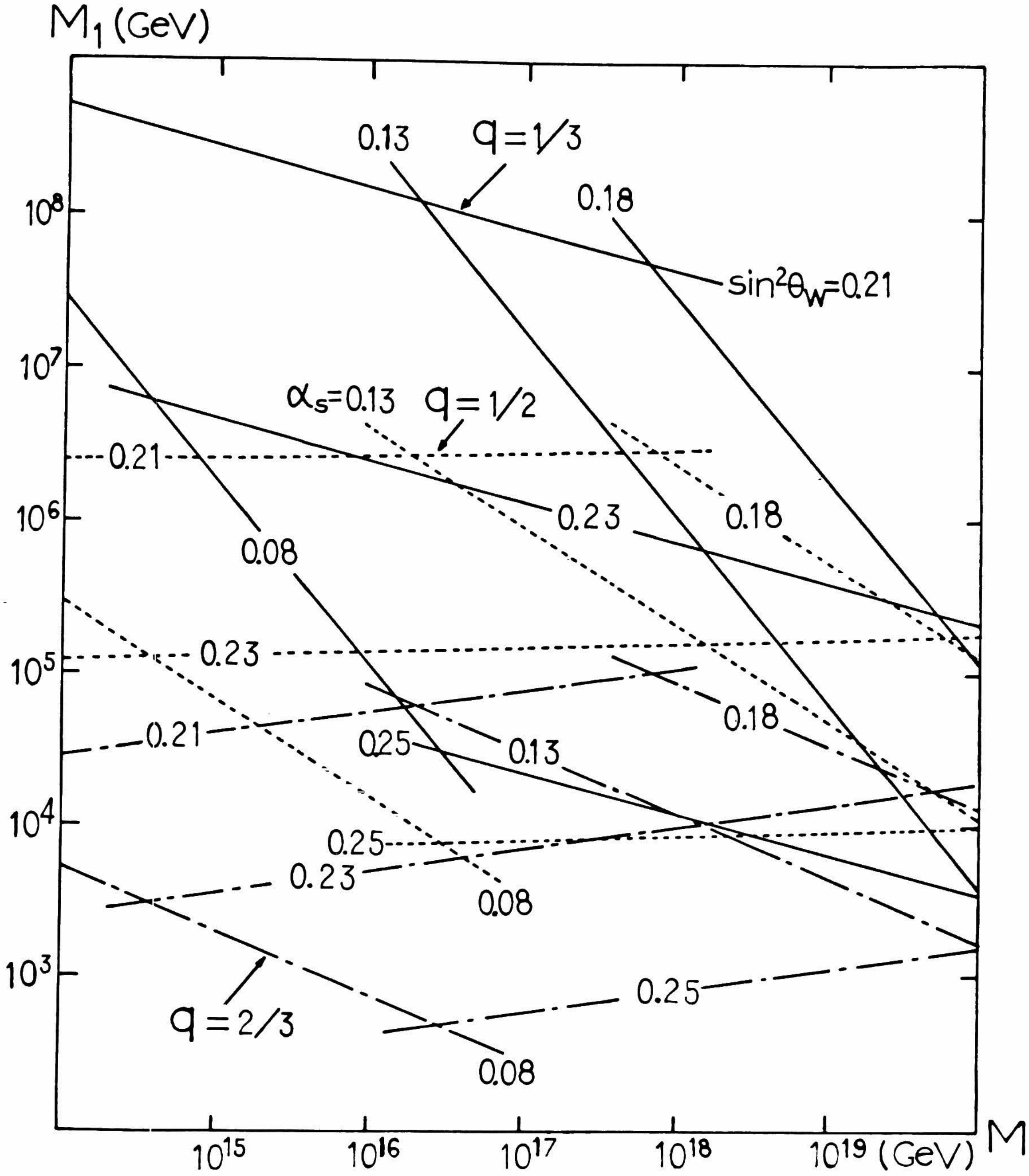

Fig.6. M-M_1 relation for the one-step enlargement (Δ_2=Δ_1) to the modified SU(N). A set of solid lines are written for q=1/3 and that of dotted ones for q=1/2 and that of dash dotted ones for q=2/3, by fixing $\sin^2\theta_W$(changing α_s) or α_s(changing $\sin^2\theta_W$).

Table III. Typical values of M and M_1 for the one-step enlargement ($\Delta_2=\Delta_1$) to the modified SU(N). Typical values are calculated for q=1/3, 1/2 and 2/3.

$\sin^2\theta_W$	α_s	M (GeV)	M_1 (GeV)		
			q=1/3	q=1/2	q=2/3
0.21	0.08	5.0×10^{12}	1.1×10^{9}	2.0×10^{6}	1.9×10^{4}
	0.13	1.9×10^{16}	1.3×10^{8}	2.6×10^{6}	6.0×10^{4}
	0.18	7.4×10^{17}	4.7×10^{7}	2.9×10^{6}	1.0×10^{5}
0.23	0.08	4.0×10^{14}	5.8×10^{6}	1.2×10^{5}	3.1×10^{3}
	0.13	1.5×10^{18}	6.5×10^{5}	1.6×10^{5}	9.6×10^{3}
	0.18	6.0×10^{19}	2.4×10^{5}	1.8×10^{5}	1.6×10^{4}
0.25	0.08	3.3×10^{16}	3.0×10^{4}	7.6×10^{3}	4.9×10^{2}
	0.13	1.2×10^{20}	3.4×10^{3}	9.8×10^{3}	1.5×10^{3}
	0.18	4.8×10^{21}	1.3×10^{3}	1.1×10^{4}	2.6×10^{3}

the model (I), we show the region compatible with the proton lifetime $\tau_p \gtrsim 10^{30}$ years ($M \gtrsim 3\times 10^{14}$GeV) calculated by Eq.(2·37) in Fig.2. One of characteristic features of the modified model (II) is that the value $\sin^2\theta_W$ of model (II) is rather close to 'experimental' one in comparison with that of model (I). If a rather large value of $\sin^2\theta_W$ should be confirmed by more accurate experiments in the future, the model (II) with $q\neq 0$ (or $\sin^2\theta_W \neq 3/8$ at M) might be of interest as an alternative to the original SU(5) model or its analogue. Although we have considered so far only one step case (L=2), features of correlation between the enlargement of the colour group and that of the flavour are expected to be similar to those of model (I), since a sum $\sum_{x=1}^{L-1} (\Delta_{x+1}-\Delta_x) t_x$ appears in Eq.(2·30) or Eq.(2·31) as well as in Eq.(2·24).

§2.4 Summary and discussion

In this section (§2), we have investigated possible structures of the gauge hierarchy from a viewpoint of 'from ground up' approach by tracing trajectories of the effective coupling constants, and derived several sum rules for the SU(N) grand unification model, which are useful for getting at features of the structure. In fact, these sum rules show clearly how the enlargement of colour group is closely correlated with that of flavour one.

(i) In the SU(N) model (I) of DG, without adjusting $\kappa_1=0$,

M_1 can be so small that new interactions would be revealed by feasible experiments if there occur two step or more enlargements of the gauge group between M_W and the unification scale M, while in the modified model (II), M_1 is small even for one step case if a suitable value for q is chosen. The latter model (II) could involve new charged quarks with charge $Q \neq n/3$ or new leptons with charge $Q=n/3$ (n:integer). If heavy particles with such an exotic charge should be produced in e^+e^- reactions etc., this might indicate the existence of new interactions, for example, of $SU(4)_C \times SU(3)_W \times U(1)$ even below a gauge threshold M_1. Detailed analyses of this model (II) (an SU(7) grand unified model) concerned with its characteristic features will be presented in succeeding sections (§§3 and 4).

(ii) As for the value $\sin^2\theta_W$ compatible with the proton lifetime (or M), the model (I) predicts somewhat smaller value ($\sin^2\theta_W \lesssim 0.22$) independently of the number of steps (L), while the model (II), for example with one step, could provide the value suitable for explaining 'experimental' $\sin^2\theta_W$. So far as we know, there are three theoretical possibilities to make $\sin^2\theta_W$ larger; (a) proliferation of SU(2) Higgs doublets in the framework of $SU(2)_L \times U(1)$ (Marciano),[26] (b) chiral extension, $SU(2)_L \to SU(2)_L \times SU(2)_R$ (e.g., Pati and Salam or SO(10) model),[20),22] (c) model with $\sin^2\theta_W \neq 3/8$ at M (e.g., modified SU(N) model), the last of which has been investigated somewhat in detail.

Throughout the analyses in this section, we have assumed for simplicity that contributions of fermions and Higgs particles can be completely eliminated ($F_1^Y = F_2^Y = F_3^Y$), higher order corrections for β-function can be neglected and mass effects near the gauge threshold are also unimportant. Main parts of our results will, however, remain unchanged even if these effects are correctly taken into account.

§3. An SU(7) Grand Unified Model and its Characteristic Features

Characteristic features of an SU(7) grand unified model are discussed, in which the fundamental representation **7** consists of SU(5)'**5** and its two singlets with the charge $q=\pm 1/2$. Fermion contents in the SU(7) and complete expressions for their mass matrices are presented. So-called survival hypothesis for fermions is naturally evaded by a kind of the electric charge conservation due to $q=\pm 1/2$, and a brief comment on the suppression of ν_{eL} mass is given also.
A possible new interaction $SU(4)_C \times SU(3)_W \times U(1)$ appears beyond an intermediate energy scale $(10^5 \sim 10^7)$GeV.

§3.1 Introduction

As is well-known, the minimal (standard) GUT, i.e., the SU(5) has very interesting features;[21),24),33)] the predicted value of $\sin^2\theta_W \simeq 0.21$, the proton decay with lifetime $\tau_p \simeq 10^{30}$ years fairly close to the present experimental lower limit, and so on. In spite of these successes, the SU(5) should be regarded as one step to understand the ultimate structure of nature since the *"mystery of generation"* remains still unsolved, which might indicate the substructure of the quark-lepton world. Furthermore, we are reluctant to accept the huge *"physical desert"* between the ordinary mass scale (M_W) and unification one (M) of the SU(5) theory. We rather may expect extended colour and/or electroweak interactions, e.g., $SU(4)_C \times SU(3)_W \times U(1)$ to appear in the near future just as the unitary flavour symmetry (although it is global) has been generally enlarged from the isospin SU(2) along with the increase of available energies. The former problems (generation problems) have been discussed by some authors enlarging the SU(5) to some larger GUT groups.[36)∿49)] As for the latter, we have investigated, in §2, several possible enlargements of 'colour' $SU(3)_C$ and 'flavour' $SU(2)_W$ in the intermediate energy region, say 10^2GeV∿10^{15}GeV, based on the SU(N) grand unification and have shown that an enlargement of the colour must be followed by another of the flavour and vice versa. Furthermore, it has been shown that if we adopt a nonzero charge assignment $\pm q$ $(0<q<1)$ for two components except **5** of the SU(5) subgroup in the fundamental representation **7** of the SU(7) GUT, the predicted value of $\sin^2\theta_W$ becomes quite consistent with the experimental one and a new interaction $SU(4)_C \times SU(3)_W \times U(1)$ appears in the region of 10^5GeV∿10^7GeV.

In this section, we take an SU(7) grand unified model, which is suggested from the general considerations in §2, as a simple possibility to attack the above problems, and analyze details of the SU(7) clarifying its characteristic features. We have been interested mainly in possibilities of new interactions in the previous section (§2). However, the enlargement of GUT group inevitably introduces more than one generation of fermions. Thus we have to do with the generation problems when we consider fermion contents of our SU(7) model. In §3.2, we discuss the contents of an anomaly free combination of fermions. The survival hypothesis[36] is naturally evaded in the anomaly free combination, which is connected with the electric charge conservation due to $q\neq 0$. Next, we exhibit complete expressions for fermion mass matrices in terms of relevant Yukawa coupling constants and vacuum expectation values of Higgs scalars, and give a brief comment on the left-handed neutrino masses (§3.3).[50] Section 3.4 is devoted to summary and discussion.

§3.2 Fermion contents in the SU(7) and evasion of the survival hypothesis

A breaking pattern in the SU(7) which we discuss here is

$$\begin{aligned} SU(7) &\to SU(4)_C\times SU(3)_W\times U(1) \text{ at } M \\ &\to SU(3)_C\times SU(2)_W\times U(1)_{1/2} \text{ at } M_1 \to SU(3)_C\times U(1)^{em}_{1/2} \text{ at } M_W . \end{aligned} \qquad (3\cdot 1)$$

For the fundamental **7**, the following hypercharge is assigned corresponding to $q=1/2$:

$$Y/2 = (\ 1/2,-1/3,-1/3,-1/3,\ 1/2,\ 1/2,-1/2)\ . \qquad (3\cdot 2)$$

Therefore the electric charge of the **7** is

$$Q = I_3+Y/2 = (\ 1/2,-1/3,-1/3,-1/3,\ 1\ ,\ 0\ ,-1/2)\ . \qquad (3\cdot3)$$

The spontaneous breaking at the first stage $M(\gtrsim 10^{15}\mathrm{GeV})$ is caused by the Higgs scalar $\phi^{\alpha}{}_{\beta}$(the regular **48** dimensional representation) and that at the second stage $M_1(10^5\mathrm{GeV}\sim 10^7\mathrm{GeV})$ by $h^{\alpha\beta}$(**21**) and $H^{\alpha\beta\gamma}{}_{\delta}$(**224**) where α,β=0∿6 and 1∿5 correspond to the indices of the SU(5). $H^{456}{}_0$ of **224** is electrically neutral only when *q=1/2* and its nonzero vacuum expectaion value (v.e.v.) keeps just the $U(1)_{Y/2}$ (Eq.(3·2)) unbroken below M_1, while h^{06} of **21** is neutral independently of the value of q. Of course we may take other values of q. In such cases, however, higher dimensional Higgs multiplets than **224** will be necessary. When q=1/3, for example, the $H^{\alpha\beta\gamma}_{\{\delta\varepsilon\}}$(**588**) (where the indices δ,ε in a curly braket are symmetric with respect to the mutual interchange, while α,β,γ without curly braket are antisymmetric) is needed for the symmetry breaking at M_1 to keep the corresponding $U(1)_{Y/2}$ unbroken since $H^{456}_{\{00\}}$ becomes electrically neutral. We fix q=1/2 for simplicity in the following investigations. Detailed analyses of the Higgs potentials and the spontaneous breakdown in the SU(7) will be discussed in §4.

We take an anomaly free combination of fermions[35),36),41)]

$$\mathbf{1} + [2] + [4] + [6] = \psi + \psi^{\alpha\beta} + \psi_{\alpha\beta\gamma} + \psi_{\alpha}\ , \qquad (3\cdot4)^{*)}$$

---

*) Note that $[4]=[3]^*$ and $[6]=[1]^*$, so $\psi^{\alpha\beta\gamma\delta}\to\psi_{\alpha\beta\gamma}$ and $\psi^{\alpha\beta\gamma\delta\varepsilon\lambda}\to\psi_{\alpha}$.

where [m] denotes the m-fold totally antisymmetric tensor which contains only **1**, **3** and $\mathbf{3}^*$ of $SU(3)_C$.[51] The combination (3·4) is unique in the SU(7) if we forbid the duplication of each representation [m],[41] and may be regarded as the spinor representation of the SO(14) into which the SU(7) is naturally embedded. The [2], [4] and [6] are decomposed with respect to the SU(5) as follows:

$$\left.\begin{aligned}
[2] &= \mathbf{1} + 2\cdot\mathbf{5} + \mathbf{10}\ , \\
[4] &= \mathbf{5}^* + \mathbf{10} + 2\cdot\mathbf{10}^*\ , \\
[6] &= 2\cdot\mathbf{1} + \mathbf{5}^*\ .
\end{aligned}\right\} \tag{3·5}$$

Who's who of the fermions contained in (3·4) is exhibited in Table IV and explicit expressions for their SU(5) decompositions (3·5) are also presented in Table V. It should be noted that the combination (3·4) includes *two* generations of mirror fermions (capital letters) which have *V+A* coupling for $SU(2)_W\times U(1)$, as well as the same number of ordinary fermions (small letters) with *V-A* coupling. In accordance with Eq.(3·3), the charges of fermions become as follows and are illustrated for clarity in Fig.7:

$$\left.\begin{aligned}
Q(u,d,\nu_e,e) &= (\ 2/3, -1/3,\ \ 0\ ,\ -1\ ), \\
Q(c,s,\nu_\mu,\mu) &= (\ 2/3, -1/3,\ \ 0\ ,\ -1\ ), \\
Q(U,D,N_E,E) &= (\ 7/6,\ 1/6,\ 1/2, -1/2), \\
Q(C,S,N_M,M) &= (\ 1/6, -5/6, -1/2, -3/2).
\end{aligned}\right\} \tag{3·6}$$

Quarks (i,j=1,2,3)

$u = \psi^{i4}$	$u^c = -\psi_{i45}$
$d = \psi^{i5}$	$d^c = -\psi_i$
$c = -\psi_{ij5}$	$c^c = \psi^{ij}$
$s = \psi_{ij4}$	$s^c = \psi_{0i6}$
$U = \psi_{ij6}$	$U^c = \psi_{0i4}$
$D = \psi^{0i}$	$D^c = -\psi_{0i5}$
$C = \psi_{0ij}$	$C^c = \psi_{i46}$
$S = \psi^{i6}$	$S^c = -\psi_{i56}$

Leptons

$\nu_e = -\psi_5$	
$e = \psi_4$	$e^c = \psi^{45}$
$\nu_\mu = \psi_{056}$	
$\mu = -\psi_{046}$	$\mu^c = -\psi_{123}$
$N_E = \psi_6$	$N_E^c = \psi^{56}$
$E = \psi_{456}$	$E^c = \psi^{46}$
$N_M = \psi_0$	$N_M^c = \psi^{05}$
$M = \psi_{045}$	$M^c = \psi^{04}$
$N = \psi$	$N^c = \psi^{06}$

Table IV. Who's who of the fermions contained in the anomaly free combination (3·4), where "c" denotes the charge conjugation.

$$[2] = \overset{\mathbf{5}}{\begin{pmatrix} M^c & \\ & D \\ N^c_M & \end{pmatrix}}_L + \overset{\mathbf{5}}{\begin{pmatrix} E^c & \\ & S \\ N^c_E & \end{pmatrix}}_L$$

$$+ \overset{\mathbf{10}}{\begin{pmatrix} & u & \\ e^c & & c^c \\ & d & \end{pmatrix}}_L + \overset{\mathbf{1}}{N^c_L}$$

$$[4] = \overset{\mathbf{5^*}}{\begin{pmatrix} \nu_\mu & \\ & s^c \\ \mu & \end{pmatrix}}_L + \overset{\mathbf{10}}{\begin{pmatrix} & c & \\ \mu^c & & u^c \\ & s & \end{pmatrix}}_L$$

$$+ \overset{\mathbf{10^*}}{\begin{pmatrix} & S^c & \\ E & & U \\ & C^c & \end{pmatrix}}_L + \overset{\mathbf{10^*}}{\begin{pmatrix} & D^c & \\ M & & C \\ & U^c & \end{pmatrix}}_L$$

$$[6] = \overset{\mathbf{5^*}}{\begin{pmatrix} \nu_e & \\ & d^c \\ e & \end{pmatrix}}_L + \overset{\mathbf{1}}{N_{EL}} + \overset{\mathbf{1}}{N_{ML}}$$

Table V. The SU(5) decompositions of [2], [4] and [6]. The suffix L denotes the left-handed Weyl spinor.

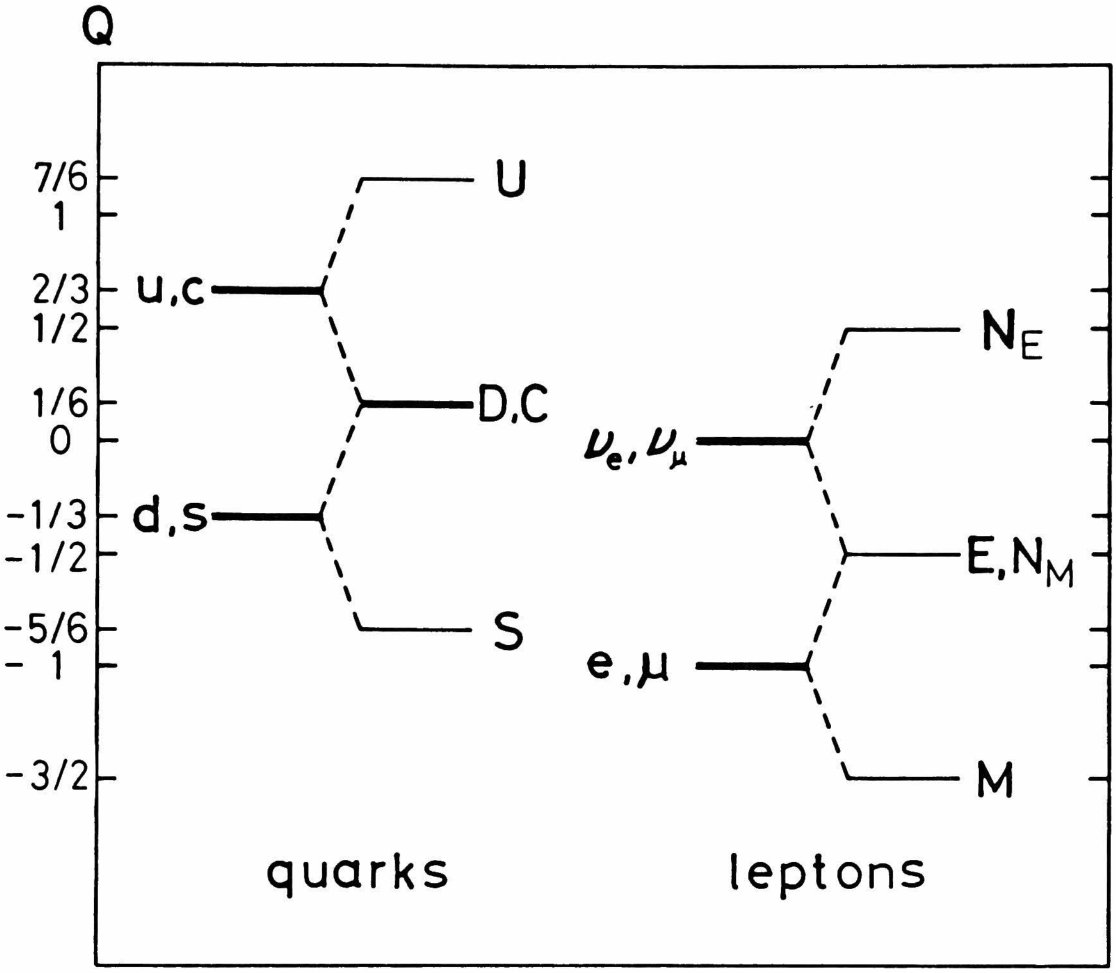

Fig.7. The charge pattern of fermions.

Spinor representations of SO(10+m) or anomaly free combinations of SU(N) which is naturally embedded into SO(2N), necessarily contain mirror fermions with V+A coupling for $SU(2)_W$. This is due to the fact that the spinor representation of SO(10+m) is decomposed into a real representation $2^{(m-1)/2}(\mathbf{16}+\mathbf{16}^*)$ for odd m or $2^{m/2-1}(\mathbf{16}+\mathbf{16}^*)$ for even m of the subgroup SO(10), where $\mathbf{16}^*$'s are the mirror fermions. Since ordinary fermions and mirror ones can form SU(5) or $SU(3)_C \times SU(2)_W \times U(1)$ invariant mass terms, they cannot survive in the low energy region ($\lesssim$several hundred GeV) according to Georgi's counting rule.[36] Because of such situation, so-called survival hypothesis, combinations (3·4) of ours were discarded in previous analyses for the generation problem based on the SU(N).[36),37] So, in such GUT's as those with multi-generations, we must think out some mechanism by virtue of which fermions become free from the survival hypothesis. We know so far, three devices;

(i) vanishing v.e.v.'s of relevant components of Higgs scalars by hand,[41),45]

(ii) technicolour models, in which ordinary fermions and mirror ones belong to different representations,[35]

(iii) a certain discrete symmetry to forbid the Yukawa couplings which mix ordinary fermions and mirror ones.[47]

At first sight our fermion combination (3·4) seems to encounter such a difficulty due to the survival hypothesis. We do not, however, need any artificial mechanism in the following way.

As is immediately seen from Fig.7, the charge patterns of ordinary fermions and mirror ones are different from each other, which forbids the dangerous mixing of them by the charge conservation. Thus we have a natural idea to liberate the fermions from the survival hypothesis by choosing boldly $q\neq 0$. Though we have illustrated this mechanism for SU(7), we will be able to apply it to such kinds of GUT's as ours. J.E. Kim proposed an SU(7) model[39] based on a similar idea to ours. He chose $q=1$, which reproduced rather small a value of $\sin^2\theta_W$ ($\lesssim 0.2$) in accordance with our previous analyses (§2).

§3.3 Mass matrices for fermions and suppression mechanism for left-handed neutrino masses

Most of the fermions acquire their masses through couplings with $SU(2)_W$ doublet Higgs scalars at the third stage of symmetry breaking M_W. However, the neutral lepton N gets a heavy Dirac mass through a coupling with **21** of SU(7) at the second stage of symmetry breaking M_1. The SU(7) multiplets of Higgs which contain $SU(2)_W$ doublets necessary for complete fermion mass generation are

$$h^{\alpha}(\mathbf{7}),\quad h^{\alpha\beta\gamma}(\mathbf{35}),\quad H^{\alpha\beta}{}_{\gamma}(\mathbf{140}), \tag{3·7}$$

and the $SU(2)_W$ doublets in these multiplets are

$$h^a,\quad h^{0a6},\quad H^{a0}{}_0,\quad H^{ai}{}_i,\quad H^{ab}{}_b,\quad H^{06}{}_a,\quad H^{a6}{}_0. \tag{3·8}$$ *)

(i=1,2,3 and a,b=4,5)

We can easily write down the most general SU(7) invariant Yukawa couplings of the fermions (3·4) with Higgs multiplets h^α, $h^{\alpha\beta\gamma}$, $H^{\alpha\beta}{}_\gamma$ and $h^{\alpha\beta}$:

$$\mathcal{L}_7 = \frac{1}{2!} f_1\psi^{\alpha\beta}\psi_{\alpha\beta\gamma}h^\gamma + f_2\psi^{\alpha\beta}\psi_\alpha h_\beta + f_3\psi\psi_\alpha h^\alpha, \tag{3·9}$$

$$\mathcal{L}_{35} = \frac{1}{2!2!3!} f_4\varepsilon_{\alpha\beta\gamma\delta\varepsilon\kappa\lambda}\psi^{\alpha\beta}\psi^{\gamma\delta}h^{\varepsilon\kappa\lambda}$$

$$+ \frac{1}{3!3!} f_5\varepsilon^{\alpha\beta\gamma\delta\varepsilon\kappa\lambda}\psi_\alpha\psi_{\beta\gamma\delta}h_{\varepsilon\kappa\lambda} + f_6\psi\psi_{\alpha\beta\gamma}h^{\alpha\beta\gamma}, \tag{3·10}$$

$$\mathcal{L}_{140} = \frac{1}{2!} f_7\psi^{\alpha\beta}\psi_{\alpha\gamma\delta}H^{\gamma\delta}{}_\beta + \frac{1}{2!} f_8\psi^{\alpha\beta}\psi_\gamma H_{\alpha\beta}{}^\gamma$$

$$+ \frac{1}{3!2!2!} f_9\varepsilon^{\alpha\beta\gamma\delta\varepsilon\kappa\lambda}\psi_{\alpha\beta\gamma}\psi_{\delta\varepsilon\rho}H_{\kappa\lambda}{}^\rho, \tag{3·11}$$

$$\mathcal{L}_{21} = \frac{1}{2!}\tilde{f}\psi\psi^{\alpha\beta}h_{\alpha\beta}. \tag{3·12}$$

By replacing the Higgs fields by their nonvanishing v.e.v.'s (where for simplicity we assume that the v.e.v.'s are all real),

---

*) Note that $H^{a6}{}_6 = -H^{a0}{}_0 - H^{ai}{}_i - H^{ab}{}_b$ since $H^{\alpha\beta}{}_\gamma$ is traceless for the contraction of upper and lower indices.

$$\left.\begin{array}{l} \langle h^{06}\rangle \equiv \tilde{V}, \\ \langle h^{5}\rangle \equiv v,\ \frac{1}{3}\langle H^{5i}{}_{i}\rangle \equiv v',\ \langle H^{54}{}_{4}\rangle \equiv v'',\ \langle H^{50}{}_{0}\rangle \equiv v_W, \\ \langle h^{056}\rangle \equiv \delta,\ \langle H^{06}{}_{5}\rangle \equiv \delta',\ \langle H^{46}{}_{0}\rangle \equiv \Delta, \end{array}\right\} \tag{3·13}$$

we obtain the following fermion mass matrices :

$$M^u = \begin{array}{c c} & \begin{array}{cc} u_0 & c_0 \end{array} \\ \begin{array}{c} u_0^c \\ c_0^c \end{array} & \left(\begin{array}{c|c} -f_1 v+f_7(v'+v'') & 2f_9\delta' \\ \hline 2f_4\delta & -f_1 v+2f_7 v' \end{array}\right) \end{array}, \tag{3·14}$$

$$M^d = \begin{array}{c c} & \begin{array}{cc} d_0 & s_0 \end{array} \\ \begin{array}{c} d_0^c \\ s_0^c \end{array} & \left(\begin{array}{c|c} -f_2 v+f_8 v' & -f_5\delta \\ \hline -f_7\delta' & 2f_9(2v'+v'') \end{array}\right) \end{array}, \tag{3·15}$$

$$M^e = \begin{array}{c c} & \begin{array}{cc} e_0 & \mu_0 \end{array} \\ \begin{array}{c} e_0^c \\ \mu_0^c \end{array} & \left(\begin{array}{c|c} f_2 v-f_8 v'' & f_7\delta' \\ \hline f_5\delta & -6f_9 v \end{array}\right) \end{array}, \tag{3·16}$$

$$
M^{\nu}=\begin{array}{c|cc|cc|}
 & \nu_{0e} & \nu_{0\mu} & N_0^c & N_0 \\
\hline
\nu_{0e} & 0 & 0 & -\frac{1}{2}f_2\delta' & -\frac{1}{2}f_3 v \\
\nu_{0\mu} & 0 & 0 & -\frac{1}{2}f_1 v - \frac{1}{2}f_7(3v'+v'') & -\frac{1}{2}f_6 v \\
N_0^c & -\frac{1}{2}f_2\delta' & -\frac{1}{2}f_1 v - \frac{1}{2}f_7(3v'+v'') & 0 & \frac{1}{2}\tilde{f}\tilde{V} \\
N_0 & -\frac{1}{2}f_3 v & -\frac{1}{2}f_6 v & \frac{1}{2}\tilde{f}\tilde{V} & 0
\end{array} , \tag{3·17}
$$

$$
M^Q = \begin{array}{c|cc|}
 & D_0 & C_0 \\
\hline
D_0^c & f_7(v_W+v')-f_1 v & -2f_9\Delta \\
C_0^c & -f_7\Delta & 2f_9(v_W+2v')
\end{array} , \tag{3·18}
$$

$$
m_U = 2f_9(v_W+v'+v'') , \tag{3·19}
$$

$$
m_S = f_7 v_W + f_1 v + f_7(2v'+v'') , \tag{3·20}
$$

$$
M^L = \begin{array}{c|cc|}
 & E & N_M \\
\hline
E^c & -f_7(v_W+3v')-f_3 v & f_8\Delta \\
N_M^c & f_7\Delta & -f_8 v_W+f_2 v
\end{array} , \tag{3·21}
$$

$$m_{N_E} = -f_8(v_W+3v'+v'') - f_5 v \ , \qquad (3\cdot 22)$$

$$m_M = -f_7(v_W+v'') + f_1 v \ . \qquad (3\cdot 23)$$

Realistic mass spectra for ordinary quarks and charged leptons and the Cabibbo angle ($\sin\theta_C$=0.23) are reproduced by suitably choosing the Yukawa coupling constants and the v.e.v.'s v, v', v'', δ and δ' of $SU(2)_W$ doublets which contribute to the ordinary fermion masses. The e-μ mixing is irrelevant in the charged weak currents since the masses of accompanying neutrinos are negligible. It should be noted that the v.e.v., $v_W \equiv \langle H^{50}{}_0 \rangle$, appears only in the diagonal mass terms of mirror fermions. This means that v_W should be much larger than v, v', v'', δ and δ' since mirror fermions are known to be heavier than at least a few tens GeV from experiments. We can summarize the role of each doublet in Eq.(3·8) as follows:

h^a, $H^{ai}{}_i$, $H^{ab}{}_b$ $\longrightarrow$ Masses of ordinary fermions,

h^{0a6}, $H^{06}{}_a$ $\longrightarrow$ Mixings among ordinary fermions,

$H^{a0}{}_0$ $\longrightarrow$ Masses of mirror fermions,

$H^{a6}{}_0$ $\longrightarrow$ Mixings among mirror fermions.

Finally, we present a brief comment on the neutrino masses. Our model contains an intermediate mass scale M_1 so that a heavy neutrino mass appears to suppress ν_{eL} and $\nu_{\mu L}$ masses sufficiently. **1** and [2] have one neutral lepton of $SU(2)_W$ singlet, say N_0 and N_0^c, respectively. They form a mass through the Yukawa coupling $\mathcal{L}_{21}$:

$$\mathcal{L}_{21} = \frac{1}{2!}\tilde{f}\cdot h_{\alpha\beta}\psi^{\alpha\beta}\psi \to \tilde{f}\langle h_{06}\rangle N_0^c N_0 \to 2A\cdot N_0^c N_0 \ . \qquad (3\cdot 24)$$

Since $\langle h_{06}\rangle$ determines the scale of M_1, this mass is very heavy. On the other hand ν_{0eL} and $\nu_{0\mu L}$ are included in [4] and [6], respectively. They obtain mass terms through mixing with N_0 and N_0^c by the spontaneous breakdown of $SU(2)_W\times U(1)$. In fact, the mass matrix (3·17) for ν_{0eL}, $\nu_{0\mu L}$, N_0^c and N_0 has the following form:

$$\begin{array}{c} \begin{array}{cccc} \nu_{0e} & \nu_{0\mu} & N_0^c & N_0 \end{array} \\ \begin{pmatrix} 0 & 0 & a & c \\ 0 & 0 & b & d \\ a & b & 0 & A \\ c & d & A & 0 \end{pmatrix} \end{array} , \qquad (3\cdot 25)$$

where

$$A \sim M_1, \quad a,b,c,d \sim O(1\text{MeV}\sim 1\text{GeV}) . \qquad (3\cdot 26)$$

Two larger eigenvalues which are nearly degenerate (opposite sign of them can be included into spinor fields) become

$$m_{\text{large}} \approx A + O(\frac{a^2}{A}) \qquad (3\cdot27)$$

and the eigenstates are approximately $(1/\sqrt{2})(N_0 \pm N_0^c)$. Two smaller eigenvalues are m_{ν_e} and m_{ν_μ},

$$m_{\nu_e},\ m_{\nu_\mu} \approx O(\frac{a^2}{A}) \ . \qquad (3\cdot28)$$

If we take $A \approx 10^7$GeV and $a,b,c,d \approx O(0.1\text{GeV})$ for example, we get

$$m_{\nu_e}, m_{\nu_\mu} \approx 0.1\text{eV} \sim 10\text{eV} \ . \qquad (3\cdot29)$$

We note that the above discussion is *independent of the value of q*.

§3.4 Summary

We have proposed an SU(7) grand unified model, where a certain anomaly free combination of fermions and a nontrivial charge assignment for them are adopted. Characteristic features of our SU(7) GUM closely connected with the choice q=1/2 have been shown as follows:

(i) The predicted value of $\sin^2\theta_W$ is quite consistent with the experimental one (0.23±0.02).

(ii) There is an oasis in the desert, i.e., a new interaction $SU(4)_C \times SU(3)_W \times U(1)$ appears in an intermediate mass scale 10^5GeV $\sim 10^7$GeV.

(iii) Two generations of ordinary and mirror fermions are included in the anomaly free combination of SU(7) which is naturally embedded into the spinor representation of SO(14).

(iv) Corresponding to $q=1/2$, mirror fermions have exotic charges in units of 1/6 and 1/2.

(v) Therefore, the survival hypothesis is evaded for the fermions in our model by virtue of the bold choice $q=1/2$.

(vi) A neutral lepton aquires a Dirac mass through the spontaneous symmetry breaking at M_1, which is heavy enough ($O(M_1)$) to suppress ν_{eL} and $\nu_{\mu L}$ masses.

When we will have reached the region between several tens GeV and a few hundred GeV in the future, new charge units other than 1/3, i.e., 1/2 and 1/6 might come in, which will prove the validity of our model. Phenomenological aspects of new interactions in our model will be discussed in §5.3.

§4. A Favourable Symmetry Breaking Pattern in the SU(7)

A pattern of symmetry breaking is investigated systematically in the SU(7) GUM, in which the fundamental representation **7** of SU(7) consists of SU(5)'**5** and its two singlets with the opposite charges $\pm q (q=1/2)$. By analyzing the Higgs potentials as well as the masses of scalar and gauge bosons at each stage of symmetry breaking, we find, among other things, following two results:

(i) A breaking path with $q=1/2$, $SU(7) \to SU(4)_C \times SU(3)_W \times U(1)$ at $M \to SU(3)_C \times SU(2)_W \times U(1)$ at $M_1 \to SU(3)_C \times U(1)^{em}$ at M_W, is indeed realized, and

(ii) the positivity of $(\text{mass})^2$'s of physical scalars under the masslessness condition for the scalars which cause the symmetry breaking at the successive stages is surely guaranteed,

in a finite range of the coupling constants of the Higgs potentials participating in whole stages.

§4.1 Introduction

In the preceding section (§3), we proposed as a simplest possibility an SU(7) grand unified model (GUM) in which the fundamental representation **7** consists of an SU(5)'**5** and two singlets with non-zero charge $\pm q$ $(0<q<1)$, and discussed its attractive features: i) the predicted value of $\sin^2\theta_W$ quite consistent with the experimental one, ii) possible new interactions $SU(4)_C \times SU(3)_W \times U(1)$ appearing beyond the intermediate energy scale $M_1 \simeq (10^5 \sim 10^7)$GeV, iii) a heavy neutrino, the mass of which is large (of order of M_1) enough to suppress ν_{eL} and $\nu_{\mu L}$ masses, iv) a natural evasion of the so-called survival hypothesis by the electric charge conservation (q=1/2), resulting in *two* generations of ordinary fermions and the same number of mirror ones with V+A coupling for the $SU(2)_W$, and v) exotic charges of 1/2 and 1/6 units for a certain anomaly free combination naturally embedded into a spionr representation of SO(14).

Now we should remember that it was crucial, in deriving the above features, to keep the weak hypercharge $U(1)_{1/2}$ (which corresponds to q=1/2 instead of q=0) unbroken below M_1. Although we can easily find the Higgs multiplets which are necessary for the desirable breakings, it is not so evident whether such breakings are actually realized by the Higgs potentials or not. So, in this section we analyze the Higgs potentials at the tree level and show that the breaking pattern corresponding to q=1/2 is indeed possible. In §4.2, we outline our scenario for *"The Breakdown in the SU(7) GUM"*. Next, using the transformation properties of Higgs scalars and gauge bosons under the subgroup at each step, which are summarized in Appendix A, we explore each step of symmetry breakings and obtain the masses of the Higgs scalars and the gauge bosons in terms of vacuum expectation values (v.e.v.'s) and coupling constants among the gauge and Higgs bosons.

Furthermore, it is shown that the positivity of (mass)^2's of physical scalars are indeed guaranteed in a finite range of the coupling constants of the Higgs potentials under the masslessness condition for the scalars which cause the symmetry breakings at the successive stages (§4.3). The last subsection (§4.4) is devoted to concluding remarks.

§4.2 Scenario for the symmetry breakings

The symmetry breaking pattern in the SU(7) GUM which we investigate is

$$
\begin{aligned}
SU(7) &\rightarrow SU(4)_C \times SU(3)_W \times U(1) \text{ at } M \\
&\rightarrow SU(3)_C \times SU(2)_W \times U(1)_{1/2} \text{ at } M_1 \rightarrow SU(3)_C \times U(1)^{em}_{1/2} \text{ at } M_W \,.
\end{aligned}
\tag{4·1}
$$

In order to realize this breaking pattern, we take the following types of SU(7) Higgs multiplets;

$$
\phi^\alpha{}_\beta(\mathbf{48}),\ h^{\alpha\beta}(\mathbf{21}),\ H^{\alpha\beta\gamma}{}_\delta(\mathbf{224}),\ H^{\alpha\beta}{}_\gamma(\mathbf{140}),\ h^{\alpha\beta\gamma}(\mathbf{35}),\ h^\alpha(\mathbf{7}). \tag{4·2}
$$

Decompositions of these multiplets for each intermediate symmetry are summarized in Appendix A. We follow the notations taken there referring to each Higgs scalar, e.g., $h^{ia}(4,3,-1/3)_1$ and $h^{i'a'}(3,2,1/6)_W$ of $h^{\alpha\beta}$ where $i=0\sim3$, $i'=1\sim3$, $a=4\sim6$, $a'=4\sim5$, and the numbers in brackets with subscripts '1' and 'W' denote $SU(4)_C \times SU(3)_W \times U(1)$ and $SU(3)_C \times SU(2)_W \times U(1)_{1/2}$ (i.e., symmetries above M_1 and M_W) quantum numbers with a suitable normalization for U(1) and $U(1)_{1/2}$, respectively.*)

As for the gauge bosons, we have **48** bosons $A^{\alpha}{}_{\beta}$ which are decomposed for each intermediate symmetry as follows:

$$A^{\alpha}{}_{\beta} = \left(\begin{array}{c|c} \underset{(15,1)_1}{\tilde{g}} + \underset{(1,1)_1}{\sqrt{\frac{3}{28}}\,\mathbf{1}\cdot B_0} & \underset{(4,3^*)_1}{\tilde{X}} \\ \hline \underset{(4^*,3)_1}{\tilde{X}^{\dagger}} & \underset{(1,8)_1}{\tilde{W}} - \underset{(1,1)_1}{\frac{4}{3}\sqrt{\frac{3}{28}}\,\mathbf{1}\cdot B_0} \end{array}\right) \tag{4·3}$$

for $SU(4)_C \times SU(3)_W \times U(1)$,

$$A^{\alpha}{}_{\beta} = \left(\begin{array}{c:c|c:c} & \underset{(3^*,1)_W}{G^{\dagger}} & \underset{(1,2)_W}{[x\ \ y]} & \underset{(1,1)_W}{u} \\ \hdashline \underset{(3,1)_W}{G} & \underset{(8,1)_W}{g} & \underset{(3,2)_W}{[X\ \ Y]} & \underset{(3,1)_W}{U} \\ \hline \underset{(1,2)_W}{\begin{bmatrix} x^{\dagger} \\ y^{\dagger} \end{bmatrix}} & \underset{(3^*,2)_W}{\begin{bmatrix} X^{\dagger} \\ Y^{\dagger} \end{bmatrix}} & \underset{(1,3)_W}{W} & \underset{(1,2)_W}{V} \\ \hdashline \underset{(1,1)_W}{u^{\dagger}} & \underset{(3^*,1)_W}{U^{\dagger}} & \underset{(1,2)_W}{V^{\dagger}} & \end{array}\right) \oplus \underset{(1,1)_W}{B \oplus B' \oplus B''} \tag{4·4}$$

---

*) As will be exhibited in §5.2, $U(1)=U(1)_{B_0}=\sqrt{3/56}(1,1,1,1,-4/3,-4/3,-4/3)$ and $U(1)_{1/2}=\sqrt{3/8}(1/2,-1/3,-1/3,-1/3,1/2,1/2,-1/2)$.

for $SU(3)_C \times SU(2)_W \times U(1)_{1/2}$, where neutral gauge bosons B, B' and B" correspond to the following diagonal generators of SU(7), respectively,

$$
\left.
\begin{array}{l}
B \sim \sqrt{\frac{3}{8}} \left(\frac{1}{2}, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, -\frac{1}{2} \right) = U(1)_{1/2}, \\[2ex]
B' \sim \frac{1}{4\sqrt{10}} \left(5, 2, 2, 2, -3, -3, -5 \right) = U(1)'_{1/2}, \\[2ex]
B'' \sim \frac{1}{\sqrt{140}} \left(5, -2, -2, -2, -2, -2, 5 \right),
\end{array}
\right\}
\tag{4·5}
$$

and B_0 is a linear combination of B, B' and B". We note here that nucleon decays are mediated by X and Y, which acquire superheavy masses ($\gtrsim 10^{15}$GeV) in the first symmetry breaking. Electric charges of $A^\alpha{}_\beta$ are also given as follows:

$$
Q(A^\alpha{}_\beta) = \left(
\begin{array}{c|ccc|cc|c}
0 & \frac{5}{6} & \frac{5}{6} & \frac{5}{6} & -\frac{1}{2} & \frac{1}{2} & 1 \\
\hline
-\frac{5}{6} & 0 & 0 & 0 & -\frac{4}{3} & -\frac{1}{3} & \frac{1}{6} \\
-\frac{5}{6} & 0 & 0 & 0 & -\frac{4}{3} & -\frac{1}{3} & \frac{1}{6} \\
-\frac{5}{6} & 0 & 0 & 0 & -\frac{4}{3} & -\frac{1}{3} & \frac{1}{6} \\
\hline
\frac{1}{2} & \frac{4}{3} & \frac{4}{3} & \frac{4}{3} & 0 & 1 & \frac{3}{2} \\
-\frac{1}{2} & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & -1 & 0 & \frac{1}{2} \\
\hline
-1 & -\frac{1}{6} & -\frac{1}{6} & -\frac{1}{6} & -\frac{3}{2} & -\frac{1}{2} & 0
\end{array}
\right), \tag{4·6}
$$

which result from the charge assignment $Q(\mathbf{7})$ for the fundamental representation $\mathbf{7}$ with q=1/2,

$$Q(\mathbf{7}) = \left(\frac{1}{2} , -\frac{1}{3} , -\frac{1}{3} , -\frac{1}{3} , 1 , 0 , -\frac{1}{2} \right).$$

Now we have all characters sufficient for the drama, *"The Breakdown in the SU(7) GUM"*. In what follows, we try to find the absolute minimum of the Higgs potentials stage after stage, supposing the hierarchies $M \gg M_1 \gg M_W$ because the total Higgs potentials are too complicated to treat simultaneously. Our scenario for this drama has three acts:

1) *Act I*

The symmetry breaking $SU(7) \to SU(4)_C \times SU(3)_W \times U(1)$ at the first stage M is caused by the Higgs scalar $\phi^\alpha{}_\beta$.[52] The Goldstone modes of $\phi^\alpha{}_\beta$ are eaten by the corresponding gauge bosons $\tilde{X}$ which acquire superheavy masses ($\sim M$) through this breaking, while the physical modes of $\phi^\alpha{}_\beta$ become superheavy at the same time. Most of the components contained in Higgs scalars, $h^{\alpha\beta}$, $H^{\alpha\beta\gamma}{}_\delta$, $H^{\alpha\beta}{}_\gamma$, $h^{\alpha\beta\gamma}$ and h^α, also obtain superheavy masses through the interactions with $\phi^\alpha{}_\beta$. We impose certain condtions on the coupling constants of these interaction potentials so that the relevant components of Higgs multiplets which are available for the successive symmetry breakings would remain massless (or nearly massless). Although such conditions are, of course, somewhat artificial, it would be beyond the scope of our present investigation to answer these so-called hierarchy problems.[33),53] Thus, only the gauge bosons, $\tilde{g}$, $\tilde{W}$, B_0 and the Higgs scalars $h^{ia}(4,3,-1/3)_1$,

$H_i(4^*,1,-5)_1$, $T_W{}^a(1,3,-4/3)_1$, $\tilde{H}^{ab}{}_c(1,6^*,-4/3)_1{}^{*)}$, h^{iab} $(4,3^*,-5/3)_1$, $h^a(1,3,-4/3)_1$ can survive after the first stage of symmetry breaking, and the gauge bosons $\tilde{X}(4,3^*,7/3)_1$ and unwanted scalars drop out of the stage by obtaining superheavy masses.

2) *Act II*

Breaking at the second stage M_1 takes place by developing nonvanishing v.e.v.'s of h^{06} of h^{ia} and $H_0(\equiv H^{456}{}_0)$ of H_i, and the $SU(4)_C\times SU(3)_W\times U(1)$ is broken down to the $SU(3)_C\times SU(2)_W$ $\times U(1)_{1/2}$. H_0 is electrically neutral only when q=1/2 and its nonzero v.e.v. keeps just $U(1)_{1/2}$(Eq.(4·5)) unbroken below M_1, while h^{06} is neutral independently of the value of q. Here, we should remark that $H^{\alpha\beta\gamma}{}_\delta(\mathbf{224})$ is indispensable to get the desired breaking path (4·1) with *q=1/2*.

As shown in §3.3, the v.e.v. of h^{06}, $\langle h^{06}\rangle$, can make the Dirac-type mass of an $SU(2)_W$-singlet neutral lepton so heavy $(\sim M_1)$ as to suppress sufficiently the masses of ordinary

---

*) $\tilde{H}^{ia}{}_b(4,8,1)_1$ and $H^{ab}{}_i(4^*,3^*,-11/3)_1$ of $H^{\alpha\beta}{}_\gamma$ also have $SU(2)_W$ doublets which may cause symmetry breaking at the third stage. $\tilde{H}^{06}{}_{a'}$ of $\tilde{H}^{ia}{}_b$ and $H^{6a'}{}_0$ of $H^{ab}{}_i$ generate the Cabibbo-like mixing of ordinary fermions and that of mirror ones, respectively. In this section we assume, for simplicity, that $\tilde{H}^{ia}{}_b$ and $H^{ab}{}_i$ do not survive and therefore such mixings vanish in the tree approximation. Detailed analyses of fermion mass matrices taking account of these doublets were presented in §3.

left-handed neutrinos. Since Yukawa couplings of fermions with $H^{\alpha\beta\gamma}{}_{\delta}$ are absent for our choice of fermion assignment, no fermion masses except that of such a heavy neutrino are generated up to the second stage. The gauge bosons, $g(8,1)_W$, $W(1,3)_W$, $B(1,1)_W$ and the weak doublets of Higgs scalars, $T_W{}^{a'}$, $\tilde{T}_W{}^{a'}$, $\tilde{H}^{a'6}{}_{6}$, $h^{0a'6}$, $h^{a'}$ survive as massless. Goldstone modes of h^{ia} and H_i are eaten by the gauge bosons $G(3,1)_W$, $V(1,2)_W$, $B'(1,1)_W$ and $B''(1,1)_W$. The rest of scalars as well as the gauge bosons G, V, B' and B" acquire heavy masses of order M_1.

The deviation of the v.e.v. of $\phi^{\alpha}{}_{\beta}$ from the $SU(4)_C \times SU(3)_W \times U(1)$ invariant value is induced through the couplings with $h^{\alpha\beta}$ and $H^{\alpha\beta\gamma}{}_{\delta}$, expressed in terms of $\langle h^{06}\rangle$ and $\langle H_0\rangle$. This deviation is, however, suppressed by a factor $M_1/M \ll 1$ compared with $\langle h^{06}\rangle$ or $\langle H_0\rangle$.*) So the effects from such a deviation could be neglected in the leading order. This kind of argument will be applied to the third stage of breaking.

3) *Act III*

Through preceding two stages of symmetry breaking, the "familiar characters" have been left massless, i.e., the colour octet gluons, g, the electroweak gauge bosons, W, B, and several weak doublets of Higgs scalars which are specific components of $H^{\alpha\beta}{}_{\gamma}$, $h^{\alpha\beta\gamma}$ and h^{α}. Thus the final part of the drama, i.e., the symmetry breaking at the third stage M_W, is played according to the "usual scenario" which is well-known.

---

*) See Eq.(4·36).

Although other multiplets than these scalar doublets may develop nonvanishing v.e.v.'s at this stage, effects of such v.e.v.'s are safely neglected by a factor of M_W/M_1 as is noticed in the end of Act II. Therefore, the $\Delta I=1/2$ rule for the weak gauge boson masses is maintained and the contributions to Majorana masses of neutrinos coming from couplings with $SU(2)_W$ singlet or triplet scalars are sufficiently small. The outline of our scenario is illustrated in Fig.8.

§4.3 Symmetry breakings and masses of Higgs and gauge bosons

Hereafter, we investigate the spontaneous breakdown in the SU(7) GUM, stage after stage in detail following the scenario outlined in the above subsection (§4.2).

1) *The first stage of symmetry breakings:*

$$SU(7) \to SU(4)_C \times SU(3)_W \times U(1) \text{ at } M$$

The Higgs potential of $\phi^\alpha{}_\beta$ is as usual given as follows:

$$V_1(\mathbf{48}) = -\frac{1}{2}\mu^2(\phi^\alpha{}_\beta\phi^\beta{}_\alpha) + \frac{1}{4}a(\phi^\alpha{}_\beta\phi^\beta{}_\alpha)^2 + \frac{1}{2}b\phi^\alpha{}_\beta\phi^\beta{}_\gamma\phi^\gamma{}_\delta\phi^\delta{}_\alpha, \tag{4·7}$$

where the cubic coupling is discarded for simplicity by imposing a discrete symmetry $\phi^\alpha{}_\beta \to -\phi^\alpha{}_\beta$. As is analyzed by Li,[52] if

$$b > 0 \quad \text{and} \quad a > -\frac{13}{42}b\ , \tag{4·8}$$

the absolute minimum of V_1 is located at the asymmetric v.e.v.

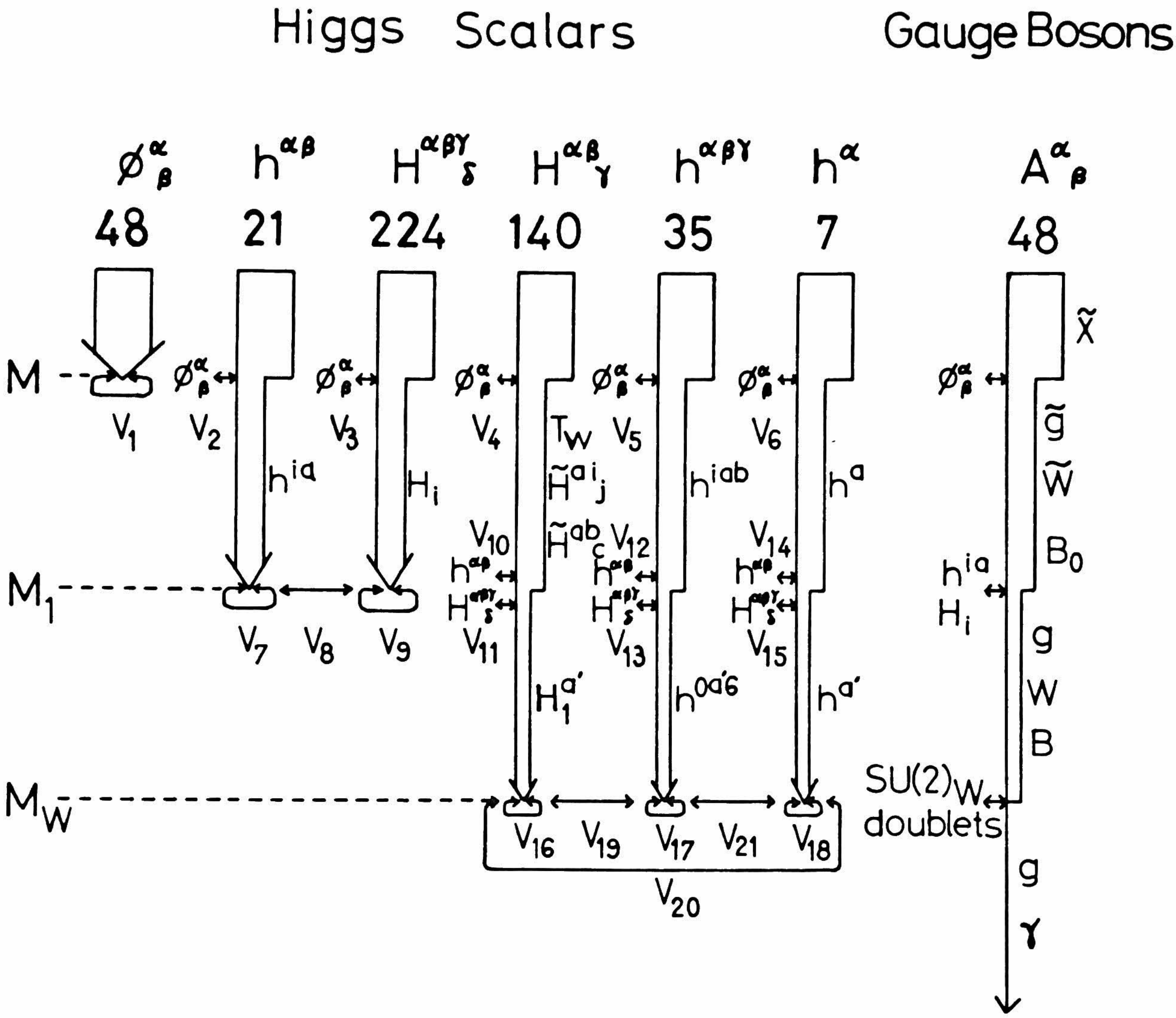

Fig.8. The outline of our scenario for *"The Breakdown in the SU(7) GUM"*. V_i's represent interaction potentials between different Higgs scalars connected by $\longleftrightarrow$ or self-interaction potentials indicated by ⮣⮢. Massless components surviving after the specific stage of symmetry breaking are shown on the right side of arrows corresponding to respective irreducible representations of SU(7).

corresponding to the desired breaking $SU(7) \to SU(4)_C \times SU(3)_W \times U(1)$.

$$\langle\phi\rangle = v \cdot \begin{pmatrix} 1 & & & & & & \\ & 1 & & & & 0 & \\ & & 1 & & & & \\ & & & 1 & & & \\ & & & & -\frac{4}{3} & & \\ & 0 & & & & -\frac{4}{3} & \\ & & & & & & -\frac{4}{3} \end{pmatrix} \qquad (4\cdot9)$$

with v determined by

$$\mu^2 = (\frac{28}{3} a + \frac{26}{9} b)\ v^2 \quad . \qquad (4\cdot10)$$

Then the gauge bosons $\tilde{X}$ have superheavy mass*)

$$m_{\tilde{X}}^2 = \frac{49}{18} g^2 v^2 \sim M \quad , \qquad (4\cdot11)$$

where g is the gauge coupling constant between Higgs scalars and gauge bosons which appears in the kinetic term of $\phi^\alpha{}_\beta$ as a covariant derivative

$$D_\mu \phi^\alpha{}_\beta = \partial_\mu \phi^\alpha{}_\beta - ig\frac{1}{\sqrt{2}}[A_\mu, \phi]^\alpha{}_\beta \ .$$

---

*) As for masses of gauge bosons, see Appendix D.

As for Higgs scalar $\phi^{\alpha}{}_{\beta}$ decomposed into $\phi_{\tilde{X}}$, $\phi_{\tilde{g}}$, $\phi_{\tilde{W}}$ and ϕ_{B_0} as $SU(4)_C\times SU(3)_W\times U(1)$ multiplets, $\phi_{\tilde{X}}$ is massless and eaten by $\tilde{X}$ gauge bosons, while the others have superheavy masses $O(M)$;

$$m^2_{\phi_{\tilde{g}}} = \frac{28}{9}\,bv^2,\quad m^2_{\phi_{\tilde{W}}} = \frac{70}{9}\,bv^2,\quad m^2_{\phi_{B_0}} = \left(\frac{56}{3}\,a + \frac{52}{9}\,b\right)v^2 = 2\mu^2, \tag{4·12}$$

which are obtained from Eq.(4·7) by substitution $\phi \to \phi + \langle\phi\rangle$. The most general couplings of $h^{\alpha\beta}$, $H^{\alpha\beta\gamma}{}_{\delta}$, $H^{\alpha\beta}{}_{\gamma}$, $h^{\alpha\beta\gamma}$ and h^{α} with $\phi^{\alpha}{}_{\beta}$ under the discrete symmetry $\phi^{\alpha}{}_{\beta} \to -\phi^{\alpha}{}_{\beta}$ are

$$V_2(\mathbf{48},\mathbf{21}) = \frac{1}{2!}\left[\alpha_1^{\phi} h^{\alpha\beta} h_{\alpha\beta}(\phi^{\gamma}{}_{\delta}\phi^{\delta}{}_{\gamma}) + \beta_1^{\phi} h^{\alpha\beta}\phi^{\gamma}{}_{\alpha}\phi^{\delta}{}_{\gamma} h_{\delta\beta} + \beta_2^{\phi} h^{\alpha\beta}\phi^{\gamma}{}_{\alpha}\phi^{\delta}{}_{\beta} h_{\gamma\delta}\right], \tag{4·13}$$

$$\begin{aligned} V_3(\mathbf{48},\mathbf{224}) = \frac{1}{3!}\Big[&\alpha_2^{\phi} H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4} H^{\alpha_4}{}_{\alpha_1\alpha_2\alpha_3}(\phi^{\gamma}_{\delta}\phi^{\delta}_{\gamma}) + \beta_3^{\phi} H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}\phi^{\beta_1}_{\alpha_1}\phi^{\beta_2}_{\alpha_2} H^{\alpha_4}{}_{\beta_1\beta_2\alpha_3} \\ &+ \beta_4^{\phi} H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}\phi^{\alpha_4}_{\alpha_3}\phi^{\beta}_{\gamma} H^{\gamma}{}_{\alpha_1\alpha_2\beta} + \beta_5^{\phi} H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}\phi^{\alpha_4}_{\beta}\phi^{\gamma}_{\alpha_3} H^{\beta}{}_{\alpha_1\alpha_2\gamma} \\ &+ \beta_6^{\phi} H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}\phi^{\beta}_{\alpha_1}\phi^{\gamma}_{\beta} H^{\alpha_4}{}_{\gamma\alpha_2\alpha_3} + \beta_7^{\phi} H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}\phi^{\alpha_4}_{\beta}\phi^{\beta}_{\gamma} H^{\gamma}{}_{\alpha_1\alpha_2\alpha_3}\Big], \end{aligned} \tag{4·14}$$

$$\begin{aligned} V_4(\mathbf{48},\mathbf{140}) = \frac{1}{2!}\Big[&\alpha_3^{\phi} H^{\alpha_1\alpha_2}{}_{\alpha_3} H^{\alpha_3}{}_{\alpha_1\alpha_2}(\phi^{\beta}_{\gamma}\phi^{\gamma}_{\beta}) + \beta_8^{\phi} H^{\alpha_1\alpha_2}{}_{\alpha_3}\phi^{\beta_1}_{\alpha_1}\phi^{\beta_2}_{\alpha_2} H^{\alpha_3}{}_{\beta_1\beta_2} \\ &+ \beta_9^{\phi} H^{\alpha_1\alpha_2}{}_{\alpha_3}\phi^{\alpha_3}_{\alpha_2}\phi^{\beta}_{\gamma} H^{\gamma}{}_{\alpha_1\beta} + \beta_{10}^{\phi} H^{\alpha_1\alpha_2}{}_{\alpha_3}\phi^{\alpha_3}_{\beta}\phi^{\gamma}_{\alpha_2} H^{\beta}{}_{\alpha_1\gamma} \\ &+ \beta_{11}^{\phi} H^{\alpha_1\alpha_2}{}_{\alpha_3}\phi^{\beta}_{\alpha_1}\phi^{\gamma}_{\beta} H^{\alpha_3}{}_{\gamma\alpha_2} + \beta_{12}^{\phi} H^{\alpha_1\alpha_2}{}_{\alpha_3}\phi^{\alpha_3}_{\beta}\phi^{\beta}_{\gamma} H^{\gamma}{}_{\alpha_1\alpha_2}\Big], \end{aligned} \tag{4·15}$$

$$\begin{aligned} V_5(\mathbf{48},\mathbf{35}) = \frac{1}{3!}\Big[&\alpha_4^{\phi} h^{\alpha\beta\gamma} h_{\alpha\beta\gamma}(\phi^{\delta}_{\varepsilon}\phi^{\varepsilon}_{\delta}) + \beta_{13}^{\phi} h^{\alpha\beta\gamma}\phi^{\delta}_{\varepsilon}\phi^{\varepsilon}_{\gamma} h_{\alpha\beta\delta} \\ &+ \beta_{14}^{\phi} h^{\alpha\beta\gamma}\phi^{\delta}_{\beta}\phi^{\varepsilon}_{\gamma} h_{\alpha\delta\varepsilon}\Big], \end{aligned} \tag{4·16}$$

$$V_6(\mathbf{48,7}) \;=\; \alpha_5^\phi h^\alpha h_\alpha(\phi^\beta_\gamma\phi^\gamma_\beta)+\beta_{15}^\phi h^\alpha\phi_\alpha^\beta\phi_\beta^\gamma h_\gamma \quad . \tag{4·17}$$

By substitution $\phi \to \phi+\langle\phi\rangle$ into Eqs.(4·13)∿(4·17), we obtain the masses of $SU(4)_C\times SU(3)_W\times U(1)$ Higgs multiplets contained in $h^{\alpha\beta}$, $H^{\alpha\beta\gamma}{}_\delta$, $H^{\alpha\beta}{}_\gamma$, $h^{\alpha\beta\gamma}$ and h^α in terms of v and relevant coupling constants, α_i^ϕ and β_i^ϕ:

$$
\begin{array}{lcllll}
h^{\alpha\beta} & : & \frac{28}{3}\alpha_1^\phi & \beta_1^\phi & \beta_2^\phi & \\
m_h^2 ij & = & (\;1 & 1 & 1 &)v^2 \\
m_h^2 ia & = & (\;1 & \frac{25}{18} & -\frac{4}{3} &)v^2 \\
m_h^2 ab & = & (\;1 & \frac{16}{9} & \frac{16}{9} &)v^2
\end{array}
\Bigg\} \;, \tag{4·18}
$$
[*)]

$$
\begin{array}{lclllllll}
H^{\alpha\beta\gamma}{}_\delta & : & \frac{28}{3}\alpha_2^\phi & \beta_3^\phi & \beta_4^\phi & \beta_5^\phi & \beta_6^\phi & \beta_7^\phi & \\
m_{\tilde{H}}^2 ijk_l & = & (\;1 & 1 & 0 & 1 & 1 & 1 &)v^2 \\
m_{\tilde{H}}^2 aij_k & = & (\;1 & -\frac{5}{9} & 0 & \frac{2}{9} & \frac{34}{27} & 1 &)v^2 \\
m_{\tilde{H}}^2 ijk_a & = & (\;1 & 1 & 0 & -\frac{4}{3} & 1 & \frac{16}{9} &)v^2 \\
m_{\tilde{H}}^2 ija_b & = & (\;1 & -\frac{5}{9} & 0 & -\frac{8}{27} & \frac{34}{27} & \frac{16}{9} &)v^2 \\
m_{\tilde{H}}^2 abi_j & = & (\;1 & -\frac{8}{27} & 0 & -\frac{5}{9} & \frac{41}{27} & 1 &)v^2 \\
m_{H_i}^2 & = & (\;1 & \frac{16}{9} & 0 & -\frac{4}{3} & \frac{16}{9} & 1 &)v^2 \\
m_{\tilde{H}}^2 iab_c & = & (\;1 & -\frac{8}{27} & 0 & -\frac{20}{27} & \frac{41}{27} & \frac{16}{9} &)v^2 \\
m_{T_C}^2 ij & = & (\;1 & \frac{17}{45} & \frac{98}{45} & \frac{13}{27} & \frac{149}{135} & \frac{59}{45} &)v^2 \\
m_{T_M}^2 ia & = & (\;1 & -\frac{2}{5} & \frac{98}{45} & \frac{8}{15} & \frac{191}{135} & \frac{22}{15} &)v^2 \\
m_{T_W}^2 ab & = & (\;1 & \frac{184}{135} & \frac{98}{135} & \frac{59}{45} & \frac{233}{135} & \frac{73}{45} &)v^2
\end{array}
\Bigg\} \;, \tag{4·19}
$$

---

*) Each line should be read as $m_h^2 ij=(28\alpha_1^\phi/3 + \beta_1^\phi + \beta_2^\phi)v^2$.

$$\left.\begin{array}{lcccccccc}
H^{\alpha\beta}{}_{\gamma} & : & \frac{28}{3}\alpha_3^{\phi} & \beta_8^{\phi} & \beta_9^{\phi} & \beta_{10}^{\phi} & \beta_{11}^{\phi} & \beta_{12}^{\phi} & \\
m^2_{\tilde{H}^{ij}{}_k} & = (& 1 & 1 & 0 & 1 & 1 & 1 &)v^2 \\
m^2_{\tilde{H}^{ai}{}_j} & = (& 1 & -\frac{4}{3} & 0 & -\frac{1}{6} & \frac{25}{18} & 1 &)v^2 \\
m^2_{H^{ij}{}_a} & = (& 1 & 1 & 0 & -\frac{4}{3} & 1 & \frac{16}{9} &)v^2 \\
m^2_{\tilde{H}^{ia}{}_b} & = (& 1 & -\frac{4}{3} & 0 & \frac{2}{9} & \frac{25}{18} & \frac{16}{9} &)v^2 \\
m^2_{H^{ab}{}_i} & = (& 1 & \frac{16}{9} & 0 & -\frac{4}{3} & \frac{16}{9} & 1 &)v^2 \\
m^2_{\tilde{H}^{ab}{}_c} & = (& 1 & \frac{16}{9} & 0 & \frac{16}{9} & \frac{16}{9} & \frac{16}{9} &)v^2 \\
m^2_{T_C{}^i} & = (& 1 & -\frac{1}{6} & \frac{49}{12} & \frac{11}{18} & \frac{43}{36} & \frac{25}{18} &)v^2 \\
m^2_{T_W{}^a} & = (& 1 & \frac{20}{27} & \frac{98}{27} & \frac{61}{54} & \frac{89}{54} & \frac{41}{27} &)v^2
\end{array}\right\} , \qquad (4\cdot 20)$$

$$\left.\begin{array}{lccccc}
h^{\alpha\beta\gamma} & : & \frac{28}{3}\alpha_4^{\phi} & \beta_{13}^{\phi} & \beta_{14}^{\phi} & \\
m^2_{h^{ijk}} & = (& 1 & 1 & 1 &)v^2 \\
m^2_{h^{ija}} & = (& 1 & \frac{34}{27} & -\frac{5}{9} &)v^2 \\
m^2_{h^{iab}} & = (& 1 & \frac{41}{27} & -\frac{8}{27} &)v^2 \\
m^2_{h^{456}} & = (& 1 & \frac{16}{9} & \frac{16}{9} &)v^2
\end{array}\right\} , \qquad (4\cdot 21)$$

$$\left.\begin{array}{lcccc}
h^{\alpha} & : & \frac{28}{3}\alpha_5^{\phi} & \beta_{15}^{\phi} & \\
m^2_{h^i} & = (& 1 & 1 &)v^2 \\
m^2_{h^a} & = (& 1 & \frac{16}{9} &)v^2
\end{array}\right\} . \qquad (4\cdot 22)$$

Most of these multiplets have superheavy masses O(M), while h^{ia}, H_i, $T_W{}^a$, $\tilde{H}^{ai}{}_j$, $\tilde{H}^{ab}{}_c$, h^{iab} and h^a should remain massless so that they could play their roles as symmetry breakers at the successive stages of breaking. Therefore, we impose the following massless conditions on the coupling constants:

$$v^{-2}m_{h}^{2}{}^{ia} = \frac{28}{3}\alpha_1^\phi + \frac{25}{18}\beta_1^\phi - \frac{4}{3}\beta_2^\phi = 0, \tag{4·23}$$

$$v^{-2}m_{H_i}^{2} = \frac{28}{3}\alpha_2^\phi + \frac{16}{9}\beta_3^\phi - \frac{4}{3}\beta_5^\phi + \frac{16}{9}\beta_6^\phi + \beta_7^\phi = 0, \tag{4·24}$$

$$v^{-2}m_{T_W}^{2}{}^{a} = \frac{28}{3}\alpha_3^\phi + \frac{20}{27}\beta_8^\phi + \frac{98}{27}\beta_9^\phi + \frac{61}{54}\beta_{10}^\phi + \frac{89}{54}\beta_{11}^\phi + \frac{41}{27}\beta_{12}^\phi = 0, \tag{4·25}$$

$$v^{-2}m_{\tilde{H}}^{2}{}^{ai}{}_{j} = \frac{28}{3}\alpha_3^\phi - \frac{4}{3}\beta_8^\phi - \frac{1}{6}\beta_{10}^\phi + \frac{25}{18}\beta_{11}^\phi + \beta_{12}^\phi = 0, \tag{4·26}$$

$$v^{-2}m_{\tilde{H}}^{2}{}^{ab}{}_{c} = \frac{28}{3}\alpha_3^\phi + \frac{16}{9}\beta_8^\phi + \frac{16}{9}\beta_{10}^\phi + \frac{16}{9}\beta_{11}^\phi + \frac{16}{9}\beta_{12}^\phi = 0, \tag{4·27}$$

$$v^{-2}m_{h}^{2}{}^{iab} = \frac{28}{3}\alpha_4^\phi + \frac{41}{27}\beta_{13}^\phi - \frac{8}{27}\beta_{14}^\phi = 0, \tag{4·28}$$

$$v^{-2}m_{h}^{2}{}^{a} = \frac{28}{3}\alpha_5^\phi + \frac{16}{9}\beta_{15}^\phi = 0. \tag{4·29}$$

Eliminating suitable coupling constants in Eqs.(4·18)∿(4·22) by the use of Eqs.(4·23)∿(4·29), we find that the following conditions are *sufficient* to ensure the positivity of the (mass)^2's of physical scalars except h^{ia}, H_i, $T_W{}^a$, $\tilde{H}^{ai}{}_j$, $\tilde{H}^{ab}{}_c$, h^{iab} and h^a:

$$h^{\alpha\beta} \;:\; -8\beta_2^\phi < \beta_1^\phi < 6\beta_2^\phi, \tag{4·30}$$

$$H^{\alpha\beta\gamma}{}_{\delta}:\; \beta_3^\phi,\ \beta_6^\phi < 0\ ,\ \beta_4^\phi,\ \beta_5^\phi,\ \beta_7^\phi > 0, \tag{4·31}$$

$$H^{\alpha\beta}{}_{\gamma} \;:\; \beta_8^\phi>0,\ \ 3\beta_{10}^\phi-8\beta_8^\phi<\beta_{11}^\phi<3\beta_{10}^\phi+6\beta_8^\phi,\ \ \beta_{11}^\phi<-4\beta_{10}^\phi-8\beta_8^\phi. \tag{4·32}$$

$$h^{\alpha\beta\gamma} \ : \ -2\beta^{\phi}_{13}+5\beta^{\phi}_{14}>0, \quad \beta^{\phi}_{13}+\beta^{\phi}_{14}<0, \quad \beta^{\phi}_{13}+8\beta^{\phi}_{14}>0, \tag{4·33}$$

$$h^{\alpha} \ : \ \beta^{\phi}_{15} < 0. \tag{4·34}$$

The regions which satisfy the conditions (4·32) and (4·33) are illustrated in Figs.9 and 10, respectively. Thus we have proved that the positivity of (mass)^2's of Higgs scalars, which drop out of the game, is indeed guaranteed in a finite range of values of the coupling constants under the massless conditions (4·23)∿(4·29). Although we have supposed in the above that all of $\tilde{H}^{ai}{}_{j}$, $T_W{}^{a}$ and $\tilde{H}^{ab}{}_{c}$ survive, it may not be the case by the reason which we will discuss in the second stage 2). Even in such cases that some of them have superheavy masses, we can also choose the values of coupling constants so as to make the (mass)^2's of the scalars irrelevant to symmetry breakings positive.

2) *The second stage of symmetry breaking:*

$$SU(4)_C\times SU(3)_W\times U(1) \rightarrow SU(3)_C\times SU(2)_W\times U(1)_{1/2} \text{ at } M_1$$

In this stage the potentials among $h^{\alpha\beta}$ and $H^{\alpha\beta\gamma}{}_{\delta}$, $V_7(\mathbf{21})$, $V_8(\mathbf{21},\mathbf{224})$ and $V_9(\mathbf{224})$, are turned on (explicit expressions of these potentials are summarized in Appendix B). Correspondingly, the vacuum will become asymmetric for $SU(4)_C\times SU(3)_W\times U(1)$ and $h^{\alpha\beta}$ and $H^{\alpha\beta\gamma}{}_{\delta}$ will have nonvanishing v.e.v.'s. We show that the absolute minimum which is connected with the desired

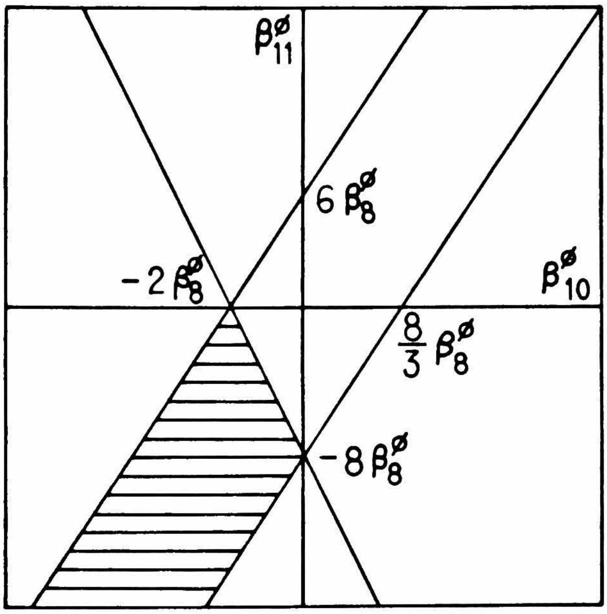

Fig.9. The allowed region by condition (4·32) in the $\beta_{10}^{\phi}-\beta_{11}^{\phi}$ plane. The shaded area represents the region allowed by Eq.(4·32) for a given $\beta_8^{\phi}>0$.

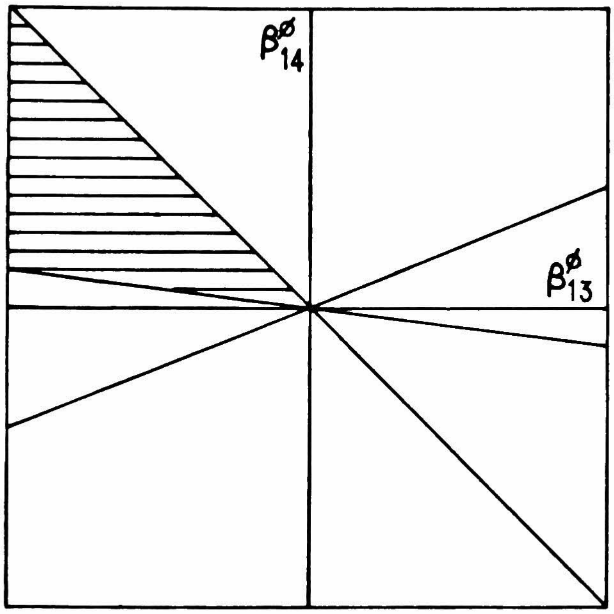

Fig.10. The allowed region by condition (4·33) in the β^{ϕ}_{13}-β^{ϕ}_{14} plane. The shaded area represents the region allowed by Eq.(4·33).

direction of the second breaking, i.e., $SU(4)_C \times SU(3)_W \times U(1) \to SU(3)_C \times SU(2)_W \times U(1)_{1/2}$, indeed exists in a finite range of the coupling constants in $V_7+V_8+V_9$. For this purpose we have only to investigate the effective $SU(4)_C \times SU(3)_W \times U(1)$ invariant potential $\tilde{V}_{eff}$ which is extracted from the combined potentials $V_1+V_2+\cdots+V_9$ so as to include only the Higgs multiplets h^{ia} and H_i surviving from the first stage of breaking, since the shifts of the minimum point of the Higgs potentials from $SU(4)_C \times SU(3)_W \times U(1)$ invariant vacuum owing to other Higgs components than h^{ia} and H_i are at most $O(M_1^2/M)$ because of their superheavy masses, and therefore negligible. These circumstances are understood in the following way as pointed out by Barbieri and Nanopoulos.[54)] We call $SU(4)_C \times SU(3)_W \times U(1)$-nonsinglet but $SU(3)_C \times SU(2)_W \times U(1)_{1/2}$-singlet components of $\phi(\mathbf{48})$ and $H(\mathbf{224})$ (or $h(\mathbf{21})$), ε and H, respectively. H is further decomposed into two classes, H' and v_1 in such a way that H' has superheavy masses through the first stage of symmetry breaking, while v_1 survives as massless. Then, each potential will have the following leading dependence on ε, H' and v_1 in the vicinity of $\phi \simeq \langle\phi\rangle$ $(4\cdot9)$:

$$\left.\begin{aligned} V_1(\phi) &\simeq \alpha\cdot v^2\varepsilon^2, \\ V_3(\phi,H) &\simeq -\beta\cdot v_1^2\varepsilon v + \beta'\cdot v^2 H'^2, \\ V_9(H) &\simeq -\frac{1}{2}\nu_H^2(v_1^2+H'^2) - \gamma_3 H' v_1^3 + \frac{1}{4}\gamma_4 v_1^4 \end{aligned}\right\} \quad (4\cdot35)$$

because of $SU(4)_C \times SU(3)_W \times U(1)$ invariance and $\phi \to -\phi$ symmetry. The factors $\alpha \cdot v^2$ and $\beta' \cdot v^2$ represent the superheavy masses of scalars and therefore α and β' are chosen to be positive. We note that the absence of the term $v^2 v_1^2$ in V_3 reflects the masslessness of v_1 at the first stage of symmetry breaking 1). The stationary conditions on ε, H' and v_1 for the combined potentials (4·35) lead to

$$\varepsilon \simeq \frac{\beta v_1}{2\alpha v} \cdot v_1 \ll v_1, \quad H' \simeq \frac{\gamma_3 v_1^2}{2\beta' v^2} \cdot v_1 \ll v_1, \tag{4·36}$$

$$v_1 \simeq \frac{\nu_H}{\sqrt{\gamma_4}} \sim O(M_1). \tag{4·37}$$

Therefore, we can conclude that the direction of breaking caused by $\tilde{V}_{eff}$ agrees with that of the whole potentials.

In accordance with the above discussion, we investigate $\tilde{V}_{eff}$ in detail. The most general form of $\tilde{V}_{eff}$ with a discrete symmetry $h \to -h$, $H \to -H$ is given by

$$\begin{aligned} \tilde{V}_{eff}(h,H) = & -\frac{1}{2}\nu_h^2 h^{ia} h_{ia} - \frac{1}{2}\nu_H^2 H^i H_i + \frac{1}{4}\tilde{\lambda}_1 (h^{ia} h_{ia})^2 \\ & + \frac{1}{4}\tilde{\lambda}_2 h^{ia} h_{ib} h^{jb} h_{ja} + \frac{1}{4}\tilde{\lambda}_3 (H^i H_i)^2 \\ & + \frac{1}{2}\tilde{\lambda}_4 h^{ia} h_{ia} H^j H_j + \frac{1}{2}\tilde{\lambda}_5 h^{ia} h_{ja} H^j H_i , \end{aligned} \tag{4·38}$$

which is also derived from the SU(7) invariant potentials $V_7+V_8+V_9$ by replacing $H^{456}{}_i$ with H_i and some linear combination of λ_j with $\tilde{\lambda}_i$ (see Appendix B). The degrees of freedom of coupling constants in the SU(7) invariant potentials $V_7+V_8+V_9$ are so sufficient that $\tilde{\lambda}_i$'s can be chosen independently each other. Now we search for the absolute minimum of $\tilde{V}_{eff}$. Firstly we vary h^{ia} and H^i in the $SU(4)_C$ space keeping $h_a \equiv (\sum_i h^{ia}h_{ia})^{1/2}$ and $H \equiv (\sum_i H^i H_i)^{1/2}$. If we assume

$$\tilde{\lambda}_2,\ \tilde{\lambda}_5 < 0 \quad , \tag{4·39}$$

$\tilde{V}_{eff}$ becomes minimum when h^{ia} and H^i lie on the same direction in the colour space, which is defined as the 0-th component of the $SU(4)_C$, *independently of a* because of $(\sum_i h^{ia}h_{ib})(\sum_j h^{jb}h_{ja}) \le (\sum_i h^{ia}h_{ia})(\sum_j h^{jb}h_{jb})$ and $(\sum_i h^{ia}H_i)(\sum_j H^j h_{ja}) \le (\sum_i h^{ia}h_{ia})(\sum_j H^j H_j)$. Then using a suitable $SU(3)_W$ rotation, we can set without loss of generality

$$|h^{06}| = (\sum_a h_a^2)^{1/2} = (h^{ia}h_{ia})^{1/2} \equiv h, \quad |H^0| = H,$$

$$\text{other components} = 0. \tag{4·40}$$

Thus, the $\tilde{V}_{eff}(h,H)$ (4·38) has the following simple form by virtue of Eq.(4·40)

$$\tilde{V}_{eff}(h,H) = -\frac{1}{2}\nu_h^2 h^2 - \frac{1}{2}\nu_H^2 H^2 + \frac{1}{4}(\tilde{\lambda}_1+\tilde{\lambda}_2)h^4$$
$$+ \frac{1}{4}\tilde{\lambda}_3 H^4 + \frac{1}{2}(\tilde{\lambda}_4+\tilde{\lambda}_5)h^2H^2 . \tag{4·41}$$

Secondly we vary h and H to find the stationary points of $\tilde{V}_{eff}$. The stationary conditions on h and H become

$$\left.\begin{aligned} \frac{\partial \tilde{V}_{eff}}{\partial h} &= \{-\nu_h^2 + (\tilde{\lambda}_1+\tilde{\lambda}_2)h^2 + (\tilde{\lambda}_4+\tilde{\lambda}_5)H^2\}\, h = 0, \\ \frac{\partial \tilde{V}_{eff}}{\partial H} &= \{-\nu_H^2 + \tilde{\lambda}_3\, H^2 + (\tilde{\lambda}_4+\tilde{\lambda}_5)h^2\}\, H = 0. \end{aligned}\right\} \qquad (4\cdot 42)$$

There are four stationary points

$$\left.\begin{aligned} &(a) \quad h = H = 0, \\ &(b) \quad h = 0,\ H = \nu_H/\sqrt{\tilde{\lambda}_3}\ , \\ &(c) \quad h = \nu_h/\sqrt{\tilde{\lambda}_1+\tilde{\lambda}_2}\ ,\ H = 0, \\ &(d) \quad h = v_1,\ H = v_1', \end{aligned}\right\} \qquad (4\cdot 43)$$

where

$$\left.\begin{aligned} v_1^2 &\equiv D^{-1}\{\tilde{\lambda}_3\ \nu_h^2 - (\tilde{\lambda}_4+\tilde{\lambda}_5)\nu_H^2\}\ , \\ v_1'^2 &\equiv D^{-1}\{-(\tilde{\lambda}_4+\tilde{\lambda}_5)\nu_h^2 + (\tilde{\lambda}_1+\tilde{\lambda}_2)\nu_H^2\}\ , \\ D &\equiv (\tilde{\lambda}_1+\tilde{\lambda}_2)\tilde{\lambda}_3 - (\tilde{\lambda}_4+\tilde{\lambda}_5)^2. \end{aligned}\right\} \qquad (4\cdot 44)$$

Furthermore we find that the conditions

$$\tilde{\lambda}_1 + \tilde{\lambda}_2 > 0,\quad \tilde{\lambda}_3 > 0,\quad D > 0, \qquad (4\cdot 45)$$

must be satisfied in order that $\tilde{V}_{eff}$ should be bounded below. If $(\tilde{\lambda}_1+\tilde{\lambda}_2)$ and $\tilde{\lambda}_3$ are sufficiently larger than other $|\tilde{\lambda}_i|$'s, v_1^2 and $v_1'^2$ are positive and the condition D>0 is also satisfied. The breaking patterns of the symmetry $S^{(2)} \equiv SU(4)_C \times SU(3)_W \times U(1)$ to which the stationary points (a)∿(d) correspond are as follows:

$$\left.\begin{array}{lll}
\text{(a)} & S^{(2)} & \text{is unbroken,} \\
\text{(b)} & S^{(2)} & \to SU(3)_C \times SU(3)_W \times \tilde{U}(1)_{1/2}, \\
\text{(c)} & S^{(2)} & \to SU(3)_C \times SU(2)_W \times U(1)_{1/2} \times U(1)'_{1/2}, \\
\text{(d)} & S^{(2)} & \to SU(3)_C \times SU(2)_W \times U(1)_{1/2},
\end{array}\right\} \qquad (4\cdot46)$$

where $\tilde{U}(1)_{1/2} = \sqrt{3/4}(1/2,-1/3,-1/3,-1/3,1/6,1/6,1/6)$ and $U(1)'_{1/2} = \sqrt{1/40}(5/2,1,1,1,-3/2-3/2,-5/2)$ (see, also various U(1)'s in §5.2). The second derivatives of $\tilde{V}_{eff}$ at each stationary point become

$$\left.\begin{array}{ll}
\text{(a)} & \dfrac{\partial^2\tilde{V}_{eff}}{\partial h^2} = -\nu_h^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial H^2} = -\nu_H^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial h \partial H} = 0, \\
\text{(b)} & \dfrac{\partial^2\tilde{V}_{eff}}{\partial h^2} = -\nu_h^2 + \dfrac{\tilde{\lambda}_4+\tilde{\lambda}_5}{\tilde{\lambda}_3}\nu_H^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial H^2} = 2\nu_H^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial h \partial H} = 0, \\
\text{(c)} & \dfrac{\partial^2\tilde{V}_{eff}}{\partial h^2} = 2\nu_h^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial H^2} = \dfrac{\tilde{\lambda}_4+\tilde{\lambda}_5}{\tilde{\lambda}_1+\tilde{\lambda}_2}\nu_h^2 - \nu_H^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial h \partial H} = 0, \\
\text{(d)} & \dfrac{\partial^2\tilde{V}_{eff}}{\partial h^2} = 2(\tilde{\lambda}_1+\tilde{\lambda}_2)v_1^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial H^2} = 2\tilde{\lambda}_3 v_1'^2, \quad \dfrac{\partial^2\tilde{V}_{eff}}{\partial h \partial H} = 2(\tilde{\lambda}_4+\tilde{\lambda}_5)v_1 v_1'.
\end{array}\right\} \qquad (4\cdot47)$$

A stationary point becomes a minimum point when

$$\frac{\partial^2\tilde{V}_{eff}}{\partial h^2}\cdot\frac{\partial^2\tilde{V}_{eff}}{\partial H^2} - \left(\frac{\partial^2\tilde{V}_{eff}}{\partial h \partial H}\right)^2 > 0 \text{ and } \frac{\partial^2\tilde{V}_{eff}}{\partial h^2} > 0.$$

In accordance with Eq.(4·47) we show, in Fig.11 for the case of $D>0$, $(\tilde{\lambda}_1+\tilde{\lambda}_2)>0$, $\tilde{\lambda}_3>0$, $(\tilde{\lambda}_4+\tilde{\lambda}_5)<0$, the phase diagram on the $\nu_h^2-\nu_H^2$ plane which represents the region where each of (a)∿(d) is realized. It should be noticed that such a minimum is absolute since the regions in the phase diagram do not overlap each other and $\tilde{V}_{eff}$ is bounded below.

The masses of gauge bosons and Higgs scalars in the symmetry $S^{(2)}$ are calculated in terms of v.e.v.'s of h and H (Eq.(4·43)) for each of breaking patterns (a)∿(d) by following usual procedures. Here we present complete discussion only for the case (d) which we have investigated in this section. The results for other cases will be obtained immediately in the similar way. Hereafter we suppose for simplicity that the v.e.v.'s of h and H are real, i.e., $\langle h^{06}\rangle=v_1$ and $\langle H^0\rangle=v_1'$ are real. Gauge bosons $G(3,1)_W$, $V(1,2)_W$, $B'(1,1)_W$ and $B''(1,1)_W$ acquire heavy masses of order M_1.*) The masses of G and V are

$$m_G^2 = \frac{1}{2}\, g^2(v_1^2+v_1'^2), \qquad m_V^2 = \frac{1}{2}\, g^2 v_1^2 \ . \tag{4·48}$$

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*) The masses of gauge bosons are obtained from either local SU(7) invariant kinetic terms $\frac{1}{2!}(\mathbf{D}^\mu\cdot\mathbf{h})^{\alpha\beta}(\mathbf{D}_\mu\cdot\mathbf{h})_{\alpha\beta}$ and $\frac{1}{3!}(\mathbf{D}^\mu\cdot\mathbf{H})^{\alpha\beta\gamma}_{\ \ \ \ \delta}(\mathbf{D}_\mu\cdot\mathbf{H})_{\alpha\beta\gamma}^{\ \ \ \ \delta}$ (factors 2! and 3! come from the anti-symmetricity for the permutation of α, β and γ), or effective $SU(4)_C\times SU(3)_W\times U(1)$ invariant ones $(\mathbf{D}^\mu\cdot\mathbf{h})^{ia}(\mathbf{D}_\mu\cdot\mathbf{h})_{ia}$ and $(\mathbf{D}^\mu\cdot\mathbf{H})^{i}(\mathbf{D}_\mu\cdot\mathbf{H})_{i}$. We have checked that both of the results coincide with each other. See also Appendix D.

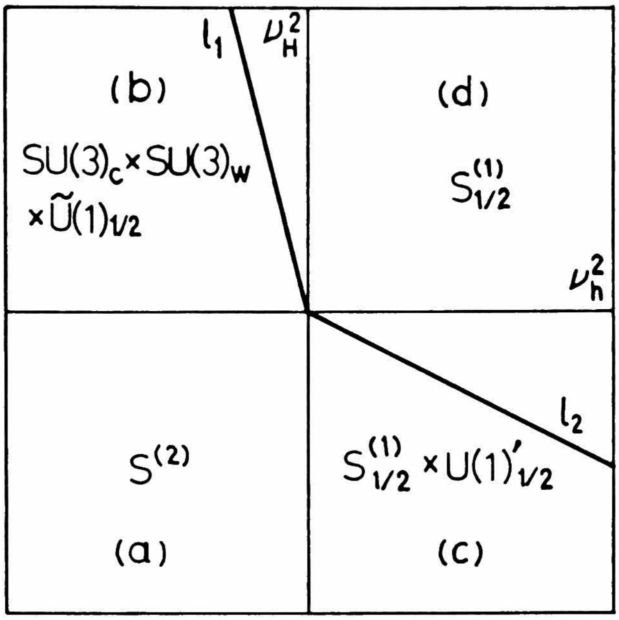

Fig.11. The phase diagram of the breaking patterns for the case $D>0$, $(\tilde{\lambda}_1+\tilde{\lambda}_2)>0$, $\tilde{\lambda}_3>0$, $(\tilde{\lambda}_4+\tilde{\lambda}_5)<0$, where $S_{1/2}^{(1)} \equiv SU(3)_C \times SU(2)_W \times U(1)_{1/2}$ and $S^{(2)} \equiv SU(4)_C \times SU(3)_W \times U(1)$. The lines l_1 and l_2 are represented by $\nu_H^2 = \tilde{\lambda}_3 \nu_h^2/(\tilde{\lambda}_4+\tilde{\lambda}_5)$ and $\nu_H^2 = (\tilde{\lambda}_4+\tilde{\lambda}_5)\nu_h^2/(\tilde{\lambda}_1+\tilde{\lambda}_2)$, respectively. For the case $(\tilde{\lambda}_4+\tilde{\lambda}_5)>0$, boundary-lines l_1 and l_2 lie in the first quadrant.

A mixing between B' and B" appears. The $(\text{mass})^2$ matrix for them is given as follows:

$$\mathcal{M}_B = 2g^2 \cdot \begin{array}{cc} B' & B'' \\ \left(\begin{array}{cc} \frac{8}{5} v_1'^2 & \frac{8}{5\sqrt{14}} v_1'^2 \\ \frac{8}{5\sqrt{14}} v_1'^2 & \frac{5}{7} v_1^2 + \frac{4}{35} v_1'^2 \end{array}\right) \end{array} . \tag{4·49}$$

The eigenvalues and the corresponding eigenstates are

$$\left.\begin{array}{ll} m^2_{\tilde{B}'} = \frac{1}{7}g^2\{(5v_1^2+12v_1'^2) - d^{1/2}\}, & \tilde{B}' = \cos\xi\cdot B' - \sin\xi\cdot B'', \\ m^2_{\tilde{B}''} = \frac{1}{7}g^2\{(5v_1^2+12v_1'^2) + d^{1/2}\}, & \tilde{B}'' = \sin\xi\cdot B' + \cos\xi\cdot B'', \end{array}\right\} \tag{4·50}$$

where

$$\left.\begin{array}{l} d = 25v_1^4 - 104v_1^2v_1'^2 + 144v_1'^4 , \\ \xi = \tan^{-1}[5\sqrt{14}\{(\frac{13}{140} - \frac{5}{112} r^{-2}) + \frac{1}{8}\sqrt{144-104r^{-2}+25r^{-4}}\}], \\ r = \dfrac{v_1'}{v_1} . \end{array}\right\} \tag{4·51}$$

Higgs multiplets $h^{0a'}$, $h^{i'a'}$, $(h^{i'6},H^{i'})$ and (h^{06},H^0) ($i'=1,2,3$ and $a'=4,5$) are treated separately each other according to the transformation properties of them with respect to the $SU(3)_C\times SU(2)_W\times U(1)_{1/2}$ which remains unbroken. The $SU(2)_W$

doublet $h^{0a'}$ is a Goldstone mode which is eaten by the gauge boson V. The mass of $h^{i'a'}$ is

$$m^2_{h^{i'a'}} = -\frac{1}{2}\tilde{\lambda}_2 v_1^2 - \frac{1}{2}\tilde{\lambda}_5 v_1'^2 . \tag{4·52}$$

The $(\text{mass})^2$ matrix for $(h^{i'6}, H^{i'})$ is

$$\mathcal{M}_3 = -\frac{1}{2}\tilde{\lambda}_5 \cdot \begin{array}{cc} h^{i'} & H^{i'} \\ \end{array} \begin{pmatrix} v_1'^2 & v_1 v_1' \\ v_1 v_1' & v_1^2 \end{pmatrix} . \tag{4·53}$$

The eigenvalues and corresponding eigenstates of $\mathcal{M}_3$ become

$$\left.\begin{aligned} & m^2_{H_3}(1) = 0, && H_3^{(1)} = \cos\theta\cdot h^{i'6} - \sin\theta\cdot H^{i'}, \\ & m^2_{H_3}(2) = -\frac{1}{2}\tilde{\lambda}_5(v_1^2+v_1'^2), && H_3^{(2)} = \sin\theta\cdot h^{i'6} + \cos\theta\cdot H^{i'}, \\ & \theta = \tan^{-1} r , && \end{aligned}\right\} \tag{4·54}$$

where $H_3^{(1)}$ is eaten by the gauge boson G. As for (h^{06}, H^0), we can treat the real parts and the imaginary ones separately since $\langle h^{06}\rangle$ and $\langle H^0\rangle$ are supposed to be real. The $(\text{mass})^2$ matrix for (Im h^{06}, Im H^0) is identically zero and suitable linear combinations of them are eaten by $\tilde{B}'$ and $\tilde{B}''$, respectively:

$$\left.\begin{aligned} H_{\tilde{B}'} &= \cos\eta\cdot \mathrm{Im}\ h^{06} - \sin\eta\cdot \mathrm{Im}\ H^0, \\ H_{\tilde{B}''} &= \sin\eta\cdot \mathrm{Im}\ h^{06} + \cos\eta\cdot \mathrm{Im}\ H^0, \\ \eta &= \tan^{-1}\left[\ \tfrac{1}{4}\{-12r+5r^{-1} + \sqrt{144r^2-104+25r^{-2}}\ \}\right]. \end{aligned}\right\} \qquad (4\cdot 55)^{*)}$$

The $(\mathrm{mass})^2$ matrix for $(\mathrm{Re}\ h^{06}, \mathrm{Re}\ H^0)$ is

$$\begin{array}{cc} \mathrm{Re}\ h^{06} & \mathrm{Re}\ H^0 \end{array}$$

$$\mathcal{M}_{\mathrm{real}} = \begin{pmatrix} (\tilde{\lambda}_1+\tilde{\lambda}_2)v_1^2 & (\tilde{\lambda}_4+\tilde{\lambda}_5)v_1v_1' \\ (\tilde{\lambda}_4+\tilde{\lambda}_5)v_1v_1' & \tilde{\lambda}_3 v_1'^2 \end{pmatrix} . \qquad (4\cdot 56)$$

The eigenvalue equation for $\mathcal{M}_{\mathrm{real}}$ is

$$t^2 - \{(\tilde{\lambda}_1+\tilde{\lambda}_2)v_1^2 + \tilde{\lambda}_3 v_1'^2\}\ t + D\ v_1^2 v_1'^2 = 0 \qquad (4\cdot 57)$$

with D defined by Eq.(4·44). The eigenvalues, say $m^2_{H_{r1}}$ and $m^2_{H_{r2}}$, are real because $\mathcal{M}_{\mathrm{real}}$ is a hermitian matrix. In order that both of them should be positive, the following conditions must be satisfied:

$$D > 0, \qquad (\tilde{\lambda}_1+\tilde{\lambda}_2)v_1^2 + \tilde{\lambda}_3 v_1'^2 > 0.$$

These inequalities are, as they should be automatically

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*) See Appendix D.

maintained by the condition (4·45).

The above results are summarized as follows: The $(\mathrm{mass})^2$'s of physical scalars are really positive by virtue of the conditions (4·40) and (4·45) for stability of the vacuum, which are of course satisfied in a finite range of the coupling constants. We demonstrate for completeness a numerical estimation of $(\mathrm{mass})^2$'s of scalars:

$$m_h^2 \text{i'a'} = 0.58\nu^2,\ m_{H_3}^2(2) = 0.33\nu^2,\ m_{H_{r1}}^2 = 2.3\nu^2,\ m_{H_{r2}}^2 = \nu^2,$$

where we have taken a particular set of values of ν_h^2, ν_H^2 and $\tilde{\lambda}_i$'s,

$$\tilde{\lambda}_1=0.15,\ \tilde{\lambda}_2= -0.05,\ \tilde{\lambda}_3=0.1,\ \tilde{\lambda}_4=\tilde{\lambda}_5= -0.02,\ \nu_h^2=\nu_H^2\equiv\nu^2.$$

Next, we discuss the fate of scalars contained in $T_W{}^a$, $\tilde{H}^{ai}{}_j$, $\tilde{H}^{ab}{}_c$, h^{iab} and h^a which have survived as massless particles since the first stage of symmetry breaking. They acquire masses in this stage through interactions with $h^{\alpha\beta}$ and $H^{\alpha\beta\gamma}{}_\delta$, $V_{10}(\mathbf{21},\mathbf{140})$, $V_{11}(\mathbf{224},\mathbf{140})$, $V_{12}(\mathbf{21},\mathbf{35})$, $V_{13}(\mathbf{224},\mathbf{35})$, $V_{14}(\mathbf{21},\mathbf{7})$ and $V_{15}(\mathbf{224},\mathbf{7})$. The most general forms of $V_{10}\sim V_{15}$ with discrete symmetries, $h^{\alpha\beta} \to -h^{\alpha\beta}$, $H^{\alpha\beta\gamma}{}_\delta \to -H^{\alpha\beta\gamma}{}_\delta$, $H^{\alpha\beta}{}_\gamma \to -H^{\alpha\beta}{}_\gamma$, $h^{\alpha\beta\gamma} \to -h^{\alpha\beta\gamma}$ and $h^\alpha \to -h^\alpha$, are presented in Appendix B. Substituting the v.e.v.'s, $\langle h^{06}\rangle=v_1$ and $\langle H^0\rangle=v_1'$, into $V_{10}+\cdots+V_{15}$, we obtain the masses of such scalars as $T_W{}^a$, $\tilde{H}^{ai}{}_j$, $\tilde{H}^{ab}{}_c$, h^{iab} and h^a, in terms of v_1, v_1' and the coupling constants, α_i^h, β_i^h, α_i^H, β_i^H in $V_{10}+\cdots+V_{15}$ under the residual symmetry $SU(3)_C\times SU(2)_W\times U(1)_{1/2}$.

The complete results are listed in Appendix C. Among these scalars, at least one of the $SU(2)_W$ doublets, $T_W{}^{a'}$, $\tilde{T}_W{}^{a'}$, $\tilde{H}^{a'6}{}_6$, $h^{0a'6}$ and $h^{a'}$,(or their suitable combinations) must survive as a massless particle after the second stage of symmetry breaking in order to play the role of the symmetry breaker in the third stage, while the unwanted scalars, e.g., $SU(2)_W$ triplet, coloured scalars etc., should have masses $O(M_1)$.

As for h^{iab} and h^a, it is easily seen from the results in Appendix C that we can make the $SU(2)_W$ doublets, $h^{0a'6}$ and $h^{a'}$ massless, while the $(\text{mass})^2$'s of other multiplets in h^{iab} and h^a all positive by suitably choosing the coupling constants, $\tilde{\alpha}_2$, $\tilde{\alpha}_3$, $\beta^h_5 \sim \beta^h_7$, $\tilde{\beta}^H_{10} \sim \tilde{\beta}^H_{16}$:

$$\left.\begin{array}{l} m^2_{h^{0a'6}} = 0, \qquad m^2_{h^{a'}} = 0, \\[2ex] m^2_{h^{045}}, \quad m^2_{h^{i'45}}, \quad m^2_{h^{i'a'6}}, \quad m^2_{h^6} \simeq O(M_1^2). \end{array}\right\} \qquad (4\cdot 58)$$

On the other hand, we should remark that all of $T_W{}^{a'}$, $\tilde{T}_W{}^{a'}$ and $\tilde{H}^{a'6}{}_6$ contained in *one and the same* multiplet $H^{\alpha\beta}{}_\gamma$ cannot simultaneously remain massless for the following reason. These doublets generally mix with each other and form a 3×3 $(\text{mass})^2$ matrix. Then six conditions on the coupling constants are required to make this matrix indentically zero. Eliminating suitable coupling constants by the use of the above conditions, we find that $m^2_{\tilde{\tilde{H}}{}^{6a'}{}_{b'}} + m^2_{\tilde{H}^{45}{}_6} = 0$ and $2m^2_{\tilde{H}^{6i'}{}_0} + m^2_{\tilde{H}^{a'i'}{}_0} + m^2_{\tilde{\tilde{H}}{}^{6i'}{}_{j'}} = 0$ independently of the values of the remaining coupling constants.

Therefore the masses of $\tilde{\tilde{H}}^{6a'}{}_{b'}$, $\tilde{H}^{45}{}_{6}$, $\tilde{H}^{6i'}{}_{0}$, $\tilde{H}^{a'i'}{}_{0}$ and $\tilde{\tilde{H}}^{6i'}{}_{j}$, as well as those of $T_W{}^{a'}$, $\tilde{T}_W{}^{a'}$ and $\tilde{H}^{a'6}{}_{6}$ become zero. Such a situation is clearly disastrous because they will develop nonvanishing v.e.v.'s in the third stage of symmetry breaking to destroy $SU(3)_C$ and/or $U(1)^{em}_{1/2}$.

Alternatively, we suppose that only a suitable combination of $T_W{}^{a'}$, $\tilde{T}_W{}^{a'}$ and $\tilde{H}^{a'6}{}_{6}$ of *one* **140**-representation can survive. For an illustration, we take

$$H_1 \equiv \sqrt{\frac{2}{5}}\, T_W{}^{a'} + \sqrt{\frac{3}{5}}\, \tilde{H}^{a'6}{}_{6} \tag{4·59}$$

as such a doublet. In this case we can realize a situation that $\tilde{H}^{ai}{}_{j}$ which contains $\tilde{T}_W{}^{a'}$ has a superheavy mass $O(M)$ in the first stage of symmetry breaking since the doublet $\tilde{T}_W{}^{a'}$ is independent of H_1. As is easily seen, the $(\text{mass})^2$ matrix of $T_W{}^{a'}$ and $\tilde{H}^{a'6}{}_{6}$ must be the following form in order that H_1 should become massless:

$$\mathcal{M}_H = v_1^2 \cdot \begin{array}{c} \begin{array}{cc} \qquad\qquad T_W{}^{a'} \qquad\qquad & \qquad\qquad \tilde{H}^{a'6}{}_{6} \end{array} \\ \begin{pmatrix} \begin{array}{l} \tilde{\alpha}_1 + \frac{5}{12}\beta^h_1 + \frac{5}{24}\beta^h_2 - \frac{1}{6}\beta^h_3 \\ + \frac{1}{12}\tilde{\beta}^H_2 - \frac{2}{3}\tilde{\beta}^H_3 + \frac{5}{3}\tilde{\beta}^H_4 \\ + \frac{1}{2}\tilde{\beta}^H_5 - \frac{2}{3}\tilde{\beta}^H_7 - \frac{4}{3}\tilde{\beta}^H_9 \end{array} & \frac{1}{\sqrt{6}}(-\beta^h_1 - \frac{1}{2}\beta^h_2 + \frac{1}{4}\beta^h_3) \\ \frac{1}{\sqrt{6}}(-\beta^h_1 - \frac{1}{2}\beta^h_2 + \frac{1}{4}\beta^h_3) & \begin{array}{l} \tilde{\alpha}_1 + \frac{1}{2}\beta^h_1 + \frac{1}{4}\beta^h_2 \\ + 2\tilde{\beta}^H_4 + 2\tilde{\beta}^H_9 \end{array} \end{pmatrix} \end{array} \propto \begin{pmatrix} 1 & -\frac{2}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} & \frac{2}{3} \end{pmatrix},$$

$$\tilde{\alpha}_1 \equiv \alpha^h_1 + \alpha^H_1 \cdot \frac{v'^2_1}{v^2_1}\,, \qquad \tilde{\beta}^H_i \equiv \beta^H_i \cdot \frac{v'^2_1}{v^2_1}\,. \tag{4·60}$$

This requires two relations among the coupling constants:

$$\left.\begin{aligned} \tilde{\alpha}_1 &= -\frac{1}{18}\tilde{\beta}^H_2 + \frac{4}{9}\tilde{\beta}^H_3 - \frac{16}{9}\tilde{\beta}^H_4 - \frac{1}{3}\tilde{\beta}^H_5 + \frac{4}{9}\tilde{\beta}^H_7 + \frac{2}{9}\tilde{\beta}^H_9 , \\ \beta^h_1 + \frac{1}{2}\beta^h_2 &= -\frac{1}{2}\beta^h_3 + \frac{1}{3}\tilde{\beta}^H_2 - \frac{8}{3}\tilde{\beta}^H_3 - \frac{4}{3}\tilde{\beta}^H_4 + 2\tilde{\beta}^H_5 - \frac{8}{3}\tilde{\beta}^H_7 - \frac{40}{3}\tilde{\beta}^H_9 . \end{aligned}\right\} \quad (4\cdot 61)$$

Taking account of these relations, we find that the positive eigenvalue of $\mathcal{M}_H$, say $m^2_{H_2}$, becomes

$$m^2_{H_2} = -\frac{1}{12}\beta^h_3 \cdot v^2_1 , \quad \beta^h_3 < 0 , \quad (4\cdot 62)$$

and that the $(\text{mass})^2$'s of other scalars of $T_W{}^a$ and $\tilde{H}^{ab}{}_c$ are

$$\left.\begin{aligned} m^2_{\tilde{H}}{}^{6a'}{}_{b'} &= \{-\frac{1}{2}\beta^h_3 - \frac{1}{2}(\beta^h_1 - \frac{1}{2}\beta^h_2)\}\cdot v^2_1 , \\ m^2_{\tilde{H}}{}^{45}{}_6 &= \{-\frac{1}{2}\beta^h_3 + \frac{1}{2}(\beta^h_1 - \frac{1}{2}\beta^h_2)\}\cdot v^2_1 , \\ m^2_{T_W}{}^6 &= \{\frac{1}{6}\beta^h_4 + (\frac{1}{8}\beta^h_3 - \frac{1}{3}\beta^h_1 + \frac{1}{3}\beta^h_2) - \frac{1}{12}\tilde{\beta}^H_1\}\cdot v^2_1 . \end{aligned}\right\} \quad (4\cdot 63)$$

If $|\beta^h_3|$ and $\beta^h_4(|\beta^h_3|<\beta^h_4)$ are sufficiently larger than other independent $|\beta^h_i|$'s and $|\tilde{\beta}^H_i|$'s, the positivity of these $(\text{mass})^2$'s are really guaranteed. We will be able to have other combination of $T_W{}^{a'}$, $\tilde{T}_W{}^{a'}$ and $\tilde{H}^{a'6}{}_6$ than H_1(Eq.(4·59)), although we have illustrated the case where a particular combination H_1 remains massless.

We give here a short comment on surviving doublets in relation to the fermion masses. If only $h^{a'}$ of **7**, $h^{0a'6}$ of **35** and a certain combination out of *one* **140** contribute to the fermion masses, the relations among them will be rather restricted, typically $m_{ordinary} \simeq m_{mirror}$. Thus we may have to introduce *3 or more* **140**'s which provide different combinations as surviving doublets to obtain realistic fermion mass spectra. For detailed discussion on fermion masses, see §3.3.

3) *The third stage of symmetry breaking:*

$$SU(3)_C \times SU(2)_W \times U(1)_{1/2} \rightarrow SU(3)_C \times U(1)^{em}_{1/2} \text{ at } M_W$$

We discuss here some aspects of the symmetry breaking $SU(2)_W \times U(1)_{1/2} \rightarrow U(1)^{em}_{1/2}$ in the presence of *several* Higgs doublets. The calculation is straightforward but somewhat complicated compared with that of the one doublet case. We prepare some **7**'s, **35**'s and **140**'s at this stage. (Refer to Appendix B for the general forms of the SU(7) invariant potentials V_{16}(**140**), V_{17}(**35**), V_{18}(**7**), V_{19}(**140**,**35**), V_{20}(**140**,**7**) and V_{21}(**35**,**7**) which are reduced to $SU(3)_C \times SU(2)_W \times U(1)_{1/2}$ invariant effective potentials for $SU(2)_W$ doublets $\mathbf{h}_i$'s.) As mentioned in the second stage of symmetry breaking, we assume that only *m* $SU(2)_W$ *doublets* coming from *m different* ***140****'s* (one $SU(2)_W$ doublet from one **140**) have survived to the third stage of symmetry breaking. The effective $SU(2)_W \times U(1)_{1/2}$ invariant potential of *n* (n>m) doublets has the following form:

$$\tilde{V}_W = -\sum_{i=1}^{n}\frac{1}{2}\nu_{Wi}^2(\mathbf{h}_i^\dagger\cdot\mathbf{h}_i) + \sum_{i=1}^{n}\frac{1}{4}\tilde{\lambda}_i^W(\mathbf{h}_i^\dagger\cdot\mathbf{h}_i)^2$$

$$+ \sum_{i\neq j}\frac{1}{4}\tilde{\lambda}_{ij}^W(\mathbf{h}_i^\dagger\cdot\mathbf{h}_i)(\mathbf{h}_j^\dagger\cdot\mathbf{h}_j) + \sum_{i\neq j}\frac{1}{4}\tilde{\lambda}_{ij}^{W'}(\mathbf{h}_i^\dagger\cdot\mathbf{h}_j)(\mathbf{h}_j^\dagger\cdot\mathbf{h}_i)$$

$$+ \sum_{i\neq j}\frac{1}{8}\{\tilde{\lambda}_{ij}^{W''}(\mathbf{h}_i^\dagger\cdot\mathbf{h}_j)^2 + \tilde{\lambda}_{ij}^{W''*}(\mathbf{h}_j^\dagger\cdot\mathbf{h}_i)^2\} \quad ,$$

$$\tilde{\lambda}_{ij} = \tilde{\lambda}_{ji} \; , \tag{4·64}$$

where we have imposed discrete symmetries $\mathbf{h}_i \to -\mathbf{h}_i$. Hereafter we assume for simplicity that $\tilde{\lambda}_{ij}^{W''}$ are real.

Firstly we vary the directions of $\mathbf{h}_i$ keeping their norms $|\mathbf{h}_i|\equiv h_i$ and minimize $\tilde{V}_W$. Noticing inequalities

$$(\mathbf{h}_i^\dagger\cdot\mathbf{h}_j)(\mathbf{h}_j^\dagger\cdot\mathbf{h}_i) \le (\mathbf{h}_i^\dagger\cdot\mathbf{h}_i)(\mathbf{h}_j^\dagger\cdot\mathbf{h}_j),$$

$$(\mathbf{h}_i^\dagger\cdot\mathbf{h}_j)^2 + (\mathbf{h}_j^\dagger\cdot\mathbf{h}_i)^2 \le 2(\mathbf{h}_i^\dagger\cdot\mathbf{h}_i)(\mathbf{h}_j^\dagger\cdot\mathbf{h}_j),$$

we find that if

$$\tilde{\lambda}_{ij}^{W'} < 0, \quad \tilde{\lambda}_{ij}^{W''} < 0, \tag{4·65}$$

the minimum of $\tilde{V}_W$ occurs when all $\mathbf{h}_i$'s align on the same direction in the $SU(2)_W$ space up to the relative sign, which essentially guarantees that $U(1)_{1/2}^{em}$ remains as an exact symmetry after the third stage of symmetry breaking. We define such a direction as the down component of the $SU(2)_W$ doublet, i.e., the 5-th component of the SU(7) in our convention. Then $\tilde{V}_W$ becomes

$$\tilde{V}_W = -\sum_{i=1}^{n}\frac{1}{2}\nu_{Wi}^2 h_i^2 + \sum_{i=1}^{n}\frac{1}{4}\tilde{\lambda}_i^W h_i^4 + \sum_{i\neq j}\frac{1}{4}(\tilde{\lambda}_{ij}^W+\tilde{\lambda}_{ij}^{W'}+\tilde{\lambda}_{ij}^{W''})h_i^2 h_j^2. \quad (4\cdot66)$$

If $\nu_{Wi}^2>0$, the absolute minimum of $\tilde{V}_W$ is located at

$$h_i^4 = 0, \quad h_i^5 = \pm v_{Wi} \neq 0 , \quad (4\cdot67)$$

where v_{Wi}^2's are determined by

$$\nu_{Wi}^2 = \tilde{\lambda}_i^W v_{Wi}^2 + \sum_{j\neq i}(\tilde{\lambda}_{ij}^W+\tilde{\lambda}_{ij}^{W'}+\tilde{\lambda}_{ij}^{W''})v_{Wj}^2 . \quad (4\cdot68)$$

Substituting these v.e.v.'s into the second derivatives of $\tilde{V}_W$ and using Eq.(4·68), we obtain three kinds of $(\mathrm{mass})^2$ matrices for scalars. These matrices correspond to the charged scalars, the imaginary part of neutral ones and the real part of neutral ones which are called $\mathcal{M}_C$, $\mathcal{M}_I$ and $\mathcal{M}_R$, respectively. Matrix elements of $\mathcal{M}_C$, $\mathcal{M}_I$ and $\mathcal{M}_R$ are

$$\left.\begin{aligned}
&(\mathcal{M}_C)_{ii} = -\sum_{i\neq j}\frac{1}{2}(\tilde{\lambda}_{ij}^{W'}+\tilde{\lambda}_{ij}^{W''})v_{Wj}^2, \quad (\mathcal{M}_C)_{ij} = \frac{1}{2}(\tilde{\lambda}_{ij}^{W'}+\tilde{\lambda}_{ij}^{W''})v_{Wi}v_{Wj}, \quad (i\neq j)\\
&(\mathcal{M}_I)_{ii} = -\sum_{i\neq j}\tilde{\lambda}_{ij}^{W''} v_{Wj}^2, \quad (\mathcal{M}_I)_{ij} = \tilde{\lambda}_{ij}^{W''} v_{Wi}v_{Wj}, \quad (i\neq j)\\
&(\mathcal{M}_R)_{ii} = \tilde{\lambda}_i^W v_{Wi}^2, \quad (\mathcal{M}_R)_{ij} = (\tilde{\lambda}_{ij}^W+\tilde{\lambda}_{ij}^{W'}+\frac{1}{2}\tilde{\lambda}_{ij}^{W''})v_{Wi}v_{Wj}. \quad (i\neq j)
\end{aligned}\right\} \quad (4\cdot69)$$

$\mathcal{M}_C$ and $\mathcal{M}_I$ have Goldstone modes which are eaten by $W^\pm$ and Z^0, respectively. In order to see these features with ease, let us set $\tilde{\lambda}^W_{ij}=\tilde{\lambda}^{W'}_{ij}=\tilde{\lambda}^{W''}_{ij}\equiv\tilde{\lambda}^W$. Then $\mathcal{M}_C$, $\mathcal{M}_I$ and $\mathcal{M}_R$ becomes as follows:

$$\left.\begin{aligned}
\mathcal{M}_C &= \mathcal{M}_I = -\tilde{\lambda}^W \begin{pmatrix} \sum\limits_{i\neq 1} v_{Wi}^2 & -v_{W1}v_{W2} & \cdots \\ -v_{W1}v_{W2} & \sum\limits_{i\neq 2} v_{Wi}^2 & \cdots \\ \vdots & & \ddots \end{pmatrix} , \\
\mathcal{M}_R &= \begin{pmatrix} \tilde{\lambda}^W_1 v_{W1}^2 & \frac{5}{2}\tilde{\lambda}^W v_{W1}v_{W2} & \cdots \\ \frac{5}{2}\tilde{\lambda}^W v_{W1}v_{W2} & \tilde{\lambda}^W_2 v_{W2}^2 & \cdots \\ \vdots & & \ddots \end{pmatrix} .
\end{aligned}\right\} \tag{4·70}$$

It is immediately shown that the Goldstone modes of $\mathcal{M}_C$ and $\mathcal{M}_I$ are proportional to

$$(v_{W1}, v_{W2}, v_{W3}, \cdots) \tag{4·71}$$

and the other eigenvalues of them are all degenerate,

$$m^2 = -\tilde{\lambda}^W \sum_{i=1}^{n} v_{Wi}^2 , \quad \tilde{\lambda}^W < 0 . \tag{4·72}$$

The positivity of these eigenvalues are automatically satisfied by condition (4·65). The eigenvalues of $\mathcal{M}_R$ are positive definite if $\tilde{\lambda}^W_i$'s are large enough compared with $|\tilde{\lambda}^W|$, which is consistent with the condition that $\tilde{V}_W$ is bounded below.

Finally the masses of W and Z bosons are

$$m_W^2 = \frac{1}{2} g^2 \sum_{i=1}^{n} v_{Wi}^2, \quad m_Z^2 = \frac{m_W^2}{\cos^2\theta_W} . \tag{4·73}$$

§4.4 Concluding remarks

We have explored the symmetry breakings stage after stage in an SU(7) grand unified model. One may worry about whether this breaking pattern is realized by Higgs potentials constructed out of suitable multiplets. In fact, even if we introduce relevant kinds of scalar multiplets which contain singlet components for a certain subgroup expected to remain unbroken, nonvanishing v.e.v.'s of the singlet components may not correspond to the absolute minimum of the potentials in any range of the coupling constants, and thus the subsymmetry cannot remain unbroken after the spontaneous breakdown. This situation may be due to the lack of relevant scalar multiplets or any more. For example, the SU(7) cannot be broken into the SU(3)×SU(2)×SU(2)×U(1)×U(1) by a regular multiplet $\phi^\alpha{}_\beta$ because the most general renormalizable potential of $\phi^\alpha{}_\beta$ including the cubic coupling admits extrema only if at most two eignevalues of the hermitian matrix $\phi^\alpha{}_\beta$ are different.[52)]

Along such a line of thought, we have presented an elaborate analysis for the Higgs potentials to show that the breaking path with q=1/2 recommended by us can indeed take place at the tree level. Generally speaking, at least one kind of multiplet will be needed for each step of symmetry breaking and its mass term, $-(\nu^2/2)\cdot h^2$, sets up the energy scale of the corresponding step. Furthermore, in correlation with these features, several multiplets must enter the game to break the GUT group down to the final goal $SU(3)_C \times U(1)^{em}$. Therefore, the potentials breaking the GUT group through several steps become too complicated to treat as a whole owing to the inflation and high dimensionality of Higgs multiplets. So, we have solved the absolute minimum problem perturbatively step by step supposing the hierarchy of symmetries which is theoretically unsolved yet but physically required. The essential point of our procedure is to prove that there is sufficient room for the degrees of freedom in the potentials *at least at the tree level* to ensure at each stage the (nearly) masslessness of the non-Goldstone scalars which play important roles in successive stages and the positivity of the $(\text{mass})^2$'s of the other unwanted scalars. We have really convinced ourselves that it is the case for the path investigated by us. Similar analyses will be applied to other breaking paths. Of course, the masslessness of the relevant scalars, in other words hierarchies among the mass scales, will be spoiled by the radiative corrections and re-finetuning of the coupling constants will be necessary order by order. It is beyond our scope to answer such problems as hierarchies. We expect that detailed analyses of Higgs potentials like ours may exclude some breaking patterns, or reveal the lack of appropriate kinds of multiplets for potentials in extended GUM's.

§5. Various Symmetry Breaking Patterns in the SU(7) Unification and their Implications

Various symmetry breaking patterns are investigated systematically in the SU(7) unification. New interactions appear at the intermediate stages of breaking. Characteristic features and phenomenological aspects of these interactions are also discussed.

§5.1 Introduction

In the previous sections (§§ 3 and 4), we have investigated thoroughly the SU(7) grand unified model with a specific breaking pattern corresponding to q=1/2, i.e., $SU(7) \rightarrow SU(4)_C \times SU(3)_W \times U(1) \rightarrow SU(3)_C \times SU(2)_W \times U(1)_{1/2} \rightarrow SU(3)_C \times U(1)^{em}_{1/2}$. We can, however, suppose other breaking patterns in the SU(7) unification and expect various kinds of extended electroweak interactions, $SU(3)_W \times U(1)$,[55] $SU(2)_W \times U(1) \times U(1)'$,[56] etc., extended colour ones, $SU(4)_C$,[57] $SU(3)_C \times SU(n)_{T.C.}$[35] ($SU(n)_{T.C.}$ represents the technicolour interactions) etc. and/or horizontal ones $SU(2)_H$[58] in each breaking path. We believe it important to explore systematically possible breaking patterns of symmetries in the SU(7) since it cannot be decided from the present experimental knowledge which breaking pattern is actually realized. These possible breaking patterns and corresponding interactions will be subject to experimental tests in the future.

In §5.2, we exhibit favourable breaking patterns in the SU(7) and clarify the roles of Higgs multiplets at each breaking step. The mass scale corresponding to each breaking step is also estimated. Next, we discuss the characteristic features and phenomenological aspects of new interactions which appear at the intermediate stages of symmetry breakings (§5.3). Finally, we present further remarks on the SU(7) unification and its breaking patterns (§5.4).

§5.2 Various symmetry breaking patterns and the corresponding mass scales and Higgs multiplets

There are many breaking patterns in the SU(7) GUT which are consistent with low energy phenomena. In the standard view we may suppose that the SU(7) is spontaneously broken into $SU(5)\times SU(2)_H$ at the first step (M) where the $SU(2)_H$ corresponds to the horizontal gauge interaction.[58] This breaking is caused by the second rank antisymmetric Higgs scalar $h^{\alpha\beta}$(**21** dimensional representation of SU(7)).[52] We call the breaking paths through the $SU(5)\times SU(2)_H$ *"standard"*. On the other hand, we have recommended another breaking path through $SU(4)_C\times SU(3)_W\times U(1)$ at the first step so as to preserve the $U(1)^{em}_{1/2}$ with q=1/2 even after the last breaking step (M_W) for the various reasons discussed in §§ 3 and 4. In this case, the Higgs scalars of the regular representation, $\phi^{\alpha}{}_{\beta}$(**48**), is needed for the first breaking. We call the paths through the $SU(4)_C\times SU(3)_W\times U(1)$

"regular".[52] Of course, "regular" paths include the case of $q=0$ as well as that of $q=1/2$. The paths through $SU(3)_C\times SU(4)_W \times U(1)$ at the first step are also possible by the $\phi^{\alpha}{}_{\beta}$. One of these paths was discussed by Claudson, Yildiz and Cox,[38] and we call them *"exotic"*. Thus the breaking patterns in the SU(7) are classified into three types, i.e., *"standard"*, *"regular"* and *"exotic"* ones.

Breaking patterns are fairly complicated as illustrated in Fig.12. Various U(1)'s appearing in Fig.12 are as follows:

$$U(1)_{B_0} = \sqrt{\tfrac{3}{56}}\left(1 , 1 , 1 , 1 , -\tfrac{4}{3} , -\tfrac{4}{3} , -\tfrac{4}{3} \right),$$

$$U(1)_{1/2} = \sqrt{\tfrac{3}{8}}\left(\tfrac{1}{2} , -\tfrac{1}{3} , -\tfrac{1}{3} , -\tfrac{1}{3} , \tfrac{1}{2} , \tfrac{1}{2} , -\tfrac{1}{2} \right),$$

$$U(1)_{0} = \sqrt{\tfrac{3}{5}}\left(0 , -\tfrac{1}{3} , -\tfrac{1}{3} , -\tfrac{1}{3} , \tfrac{1}{2} , \tfrac{1}{2} , 0 \right),$$

$$U(1)'_{1/2} = \sqrt{\tfrac{1}{40}}\left(\tfrac{5}{2} , 1 , 1 , 1 , -\tfrac{3}{2} , -\tfrac{3}{2} , -\tfrac{5}{2} \right),$$

$$U(1)'_{0} = \tfrac{1}{2}\left(1 , 0 , 0 , 0 , 0 , 0 , -1 \right),$$

$$\tilde{U}(1)_{1/2} = \sqrt{\tfrac{3}{4}}\left(\tfrac{1}{2} , -\tfrac{1}{3} , -\tfrac{1}{3} , -\tfrac{1}{3} , \tfrac{1}{6} , \tfrac{1}{6} , \tfrac{1}{6} \right),$$

$$\tilde{U}(1)_{0} = \sqrt{\tfrac{3}{4}}\left(0 , -\tfrac{1}{3} , -\tfrac{1}{3} , -\tfrac{1}{3} , \tfrac{1}{3} , \tfrac{1}{3} , \tfrac{1}{3} \right),$$

$$\tilde{U}(1)'_{0} = \sqrt{\tfrac{1}{24}}\left(1 , 1 , 1 , 1 , -2 , -2 , 0 \right).$$

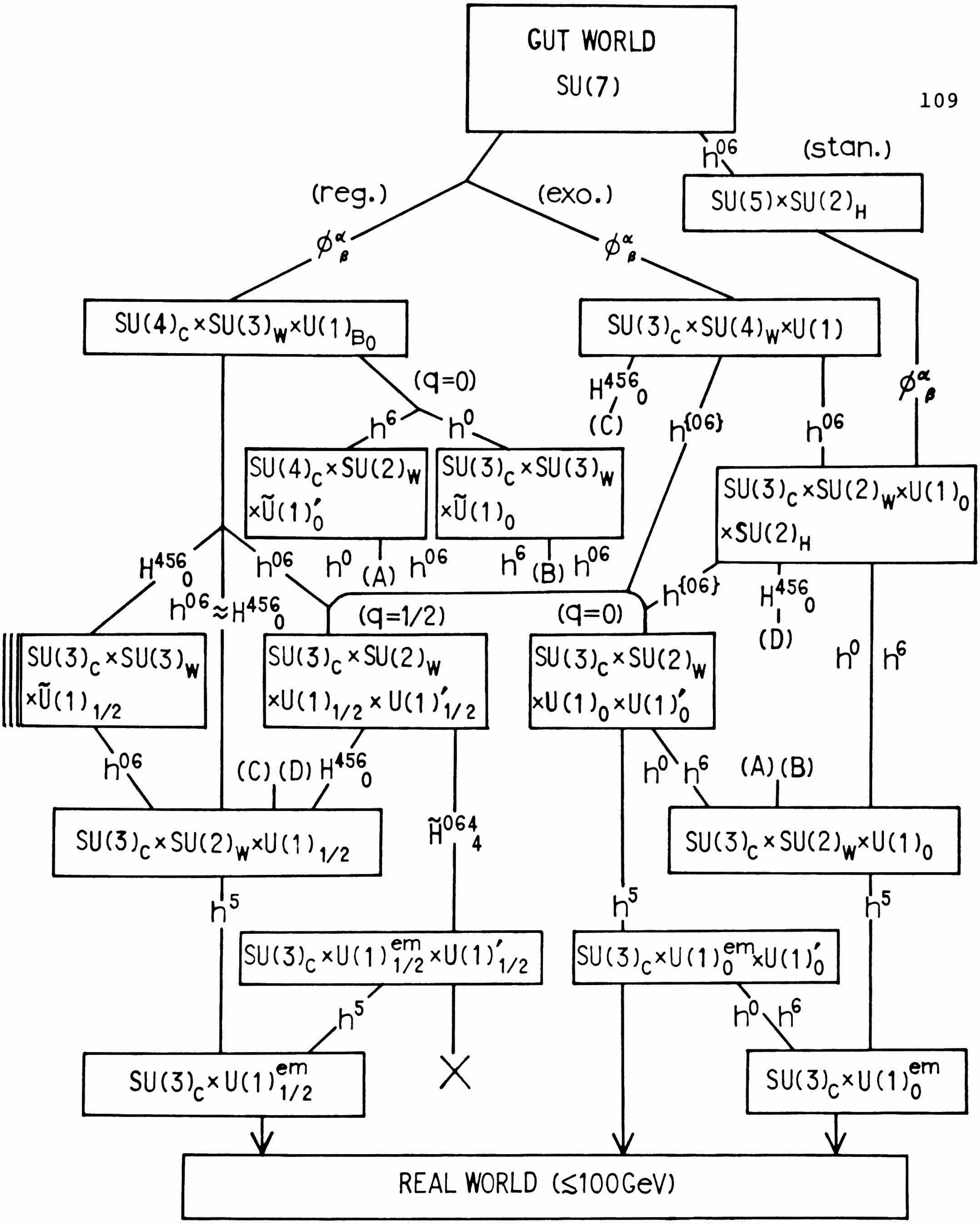

Fig.12. Breaking patterns in the SU(7) GUT and relevant components of Higgs multiplets contributing to symmetry breakings at each step. All Higgs scalars except $h^{\{06\}}$ are antisymmetric tensors with respect to upper indices, while $h^{\{06\}}$ is a symmetric tensor. The upper (A)∿(D) should be connected with the lower (A)∿(D) by straight lines, respectively.

The path through which symmetry breakings actually occur is determined by both the transformation properties of Higgs multiplets entering the game and the scales of their nonvanishing v.e.v.'s. Relevant components of Higgs multiplets which contribute to each step of symmetry breakings are also indicated in Fig.12. These breaking paths may meet and leave each other at intermediate steps and terminate in one of three possible exact symmetries which remains unbroken in our low-energy world. Among these exact symmetries, the $SU(3)_C \times U(1)_0^{em}$ is simple and conservative, while other two are somewhat radical. As one of two possibilities, we have already in §§ 3 and 4 exhibited attractive features of the breaking path terminating in the $SU(3)_C \times U(1)_{1/2}^{em}$. Another symmetry is as follows: *If another $U(1)_0'$ in addition to $U(1)_0^{em}$ remains unbroken, the survival hypothesis can be evadded even for the case of $q=0$ due to the conservation of $U(1)_0'$ charge.* Then, we have a new exactly massless gauge boson, *"techniphoton"*, suggested by Barr and Zee,[56] to which ordinary fermions can couple via a loop involving super massive gauge bosons. This device to evade the survival hypothesis is close to that of the technicolour model, but we need only a U(1) rather than some larger group. We note, however, that $U(1)_{1/2}'$ with q=1/2 cannot be exact because known fermions have consequently nonzero $U(1)_{1/2}'$ charges.

Next, we give estimates on the scales where intermediate new interactions appear. In our previous analysis (§2), we obtained some constraints on such scales by considering the renormalization effects to the GUT values of gauge coupling constants and $\sin^2\theta_W$. Among other things, the first breaking of SU(7) must occur at the scale larger than 10^{15}GeV for all kinds of paths so as to be consistent with the proton lifetime. Firstly, since "standard" paths go through the SU(5), the energy scale where $SU(5)\times SU(2)_H \to SU(3)_C\times SU(2)_W\times U(1)\times SU(2)_H$ occurs, is also larger than 10^{15}GeV. So far as the $SU(2)_H$ is electrically neutral, that is, no generators of $SU(2)_H$ contribute to the electric charge, we cannot by our previous consideration have any strict constraints on the scale where $SU(2)_H$ is broken, and may expect that flavour changing neutral current processes owing to the $SU(2)_H$ horizontal interaction will become important in 10^5GeV region.[58] On the other hand, charged $SU(2)_H$ cases with q=1/2, i.e., $SU(3)_C\times SU(2)_W\times U(1)_0\times SU(2)_H\to SU(3)_C\times SU(2)_W \times U(1)_{1/2}$, seem to be impossible because the constraint $\sin^2\theta_W \gtrsim 0.2$ requires $\alpha_s \lesssim 0.065$ for the standard paths and $M_{GUT}\lesssim 2.7\times 10^{13}$GeV for the exotic ones, respectively. Secondly, noting the fact that the extended 'flavour' $SU(4)_W$ is larger than the 'colour' $SU(3)_C$ (only the 'flavour' interaction is extended from $SU(3)_C \times SU(2)_W\times U(1)$) in the case of "exotic" paths, we find that the energy scale where $SU(3)_C\times SU(4)_W\times U(1)$ is broken to $SU(3)_C\times SU(2)_W \times U(1)\times G$ ($G=SU(2)_H$ or $U(1)$) is fairly large ($\gtrsim 10^{12}$GeV) and α_s (at 100GeV) must be bounded above ($\alpha_s \lesssim 0.01$). Finally, the extended gauge symmetry $SU(4)_C\times SU(3)_W\times U(1)$ lying on the "regular" path through the $SU(3)_C\times SU(2)_W\times U(1)_{1/2}$ with q=1/2 which we have recommended, is restored above 10^5GeV. Typical paths, energy scales and corresponding Higgs components are summarized below where use are made of the following abbreviations:

$$S^1_{1/2} \equiv SU(3)_C\times SU(2)_W\times U(1)_{1/2}, \qquad S^0_{1/2} \equiv SU(3)_C\times U(1)^{em}_{1/2},$$

$$S^1_0 \equiv SU(3)_C\times SU(2)_W\times U(1)_0, \qquad S^0_0 \equiv SU(3)_C\times U(1)^{em}_0 .$$

1) The "standard" paths

$$SU(7) \xrightarrow[h^{06}]{\gtrsim 10^{15}\mathrm{GeV}} SU(5)\times SU(2)_H \xrightarrow[\phi^{\alpha}_{\beta}]{\gtrsim 10^{15}\mathrm{GeV}} S^1_0\times SU(2)_H \longrightarrow \text{a),b)}$$

a) q=1/2 (charged $SU(2)_H$)

$$\xrightarrow[H^{456}_0]{\sim 10^2\mathrm{GeV}} S^1_{1/2}[\times U(1)'_{1/2}] \xrightarrow[h^5]{\sim 10^2\mathrm{GeV}} S^0_{1/2}$$

b) q=0 (neutral $SU(2)_H$)

$$\xrightarrow[\substack{h^0,h^6\\ [h^{\{06\}}]}]{\gtrsim 10^5\mathrm{GeV}} S^1_0[\times U(1)'_0] \xrightarrow[h^5]{\qquad} S^0_0[\times U(1)'_0]$$

2) The "regular" paths

$$SU(7) \xrightarrow[\phi^{\alpha}_{\beta}]{\gtrsim 10^{15}\mathrm{GeV}} SU(4)_C\times SU(3)_W\times U(1) \longrightarrow \text{a),b),c),d),e)}$$

a) q=1/2-I

$$\xrightarrow[H^{456}_0]{10^{9\sim 5}\mathrm{GeV}} SU(3)_C\times SU(3)_W\times \tilde{U}(1)_{1/2} \xrightarrow[h^{06}]{10^{7\sim 3}\mathrm{GeV}} S^1_{1/2} \xrightarrow[h^5]{} S^0_{1/2}$$

b) q=1/2-II

$$\xrightarrow[h^{06},H^{456}_0]{10^{7\sim 5}\mathrm{GeV}} S^1_{1/2} \xrightarrow[h^5]{} S^0_{1/2}$$

c) q=0-I

$$\xrightarrow[h^0,h^6\ [h^{06}]]{\text{no constraints}} S_0^1[\times U(1)_0'] \xrightarrow[h^5]{} S_0^0[\times U(1)_0']$$

d) q=0-II

$$\xrightarrow[h^6]{\gtrsim 10^3\text{GeV}} SU(4)_C\times SU(2)_W\times\tilde{U}(1)_0' \xrightarrow[h^0,h^{06}]{\gtrsim 10^2\text{GeV}} S_0^1 \xrightarrow[h^5]{} S_0^0$$

e) q=0-III

$$\xrightarrow[h^0]{\gtrsim 10^3\text{GeV}} SU(3)_C\times SU(3)_W\times\tilde{U}(1)_0 \xrightarrow[h^6,h^{06}]{\gtrsim 10^2\text{GeV}} S_0^1 \xrightarrow[h^5]{} S_0^0$$

3) The "exotic" paths

$$SU(7)\xrightarrow[\phi^\alpha{}_\beta]{\gtrsim 10^{15}\text{GeV}} SU(3)_C\times SU(4)_W\times U(1) \longrightarrow \text{a),b)}$$

a) q=1/2 (charged $SU(2)_H$)

$$\xrightarrow[h^{06}]{\gtrsim 10^{15}\text{GeV}} S_0^1\times SU(2)_H \xrightarrow[H^{456}_0]{\sim 10^2\text{GeV}} S_{1/2}^1 \xrightarrow[h^5]{} S_{1/2}^0$$

b) q=0 (neutral $SU(2)_H$)

$$\xrightarrow[h^{06}]{\gtrsim 10^{12}\text{GeV}} S_0^1\times SU(2)_H[\xrightarrow[h^{\{06\}}]{} S_0^1\times U(1)_0']$$

$$\xrightarrow[h^0,h^6,h^{06}]{\gtrsim 10^5\text{GeV}} S_0^1[\times U(1)_0'] \xrightarrow[h^5]{} S_0^0[\times U(1)']$$

We have investigated the path 2-b) thoroughly in §§ 3 and 4. Each path has of course its own interesting features.

§5.3 New interactions in the intermediate region and their phenomenological aspects

As has been shown in the preceding subsection (§5.2), various new interactions can be expected in the intermediate region between unification scale (M) and ordinary one (M_W) on the basis of the SU(7) unification. Roughly classifying, there are four kinds of such interactions, i.e., extended 'colour' $SU(4)_C$, extended 'flavour' $SU(3)_W$ or $SU(4)_W$, various $U(1)$'s and the 'horizontal' $SU(2)_H$. The horizontal symmetries have been already investigated by several authors. So, we discuss the characteristic features of the other three kinds of interactions in the following.

1) *Extended 'colour' $SU(4)_C$*

Implications of the extended $SU(4)_C$ were argued by several authors. In the $SU(4)\times SU(2)\times U(1)$ model, Georgi and Machacek[57] pointed out the baryon number nonconserving decay of heavy mesons which are composed of ordinary and new quarks. These decays are caused by the $SU(3)_C$ triplet gluons, say G in the $SU(4)_C$ gauge bosons, through the subprocesses $Q\rightarrow G+\bar{q}$ and $G\rightarrow\bar{q}+\bar{L}$, where q, Q and L are ordinary quarks, heavy quarks and heavy leptons, respectively. These heavy quarks and leptons can be assigned to the mirror fermions in our anomaly free SU(7) combination of fermions. The fermion assignment in the $SU(4)_C$ is as follows:

4's

$$\begin{pmatrix} M^c \\ \hdashline u \end{pmatrix}_{L'} \begin{pmatrix} N^c_M \\ \hdashline d \end{pmatrix}_{L'} \begin{pmatrix} \mu^c \\ \hdashline U \end{pmatrix}_{L'} \begin{pmatrix} N^c \\ \hdashline D \end{pmatrix}_{L'} \Big\} \; SU(3)_C$$

4*'s

$$\left(\begin{array}{c}\mu \\ \hline U^c\end{array}\right)_{L}, \quad \left(\begin{array}{c}\nu_\mu \\ \hline D^c\end{array}\right)_{L}, \quad \left(\begin{array}{c}M \\ \hline u^c\end{array}\right)_{L}, \quad \left(\begin{array}{c}N_M \\ \hline d^c\end{array}\right)_{L}, \quad \Big\} \; SU(3)_C$$

6's

$$[\, c \,,\, s^c \,]_L, \quad [\, s \,,\, c^c \,]_L, \quad [\, C \,,\, s^c \,]_L, \quad [\, S \,,\, c^c \,]_L,$$

1's

$$\nu_{eL},\ N_L,\ e_L,\ e_L^c,\ N_{EL},\ N_{EL}^c,\ E_L,\ E_L^c.$$

Besides the baryon number nonconservation, we have also B-L violating processes by the spontaneous breakdown of the $SU(4)_C$. This situation may be understood as follows. The $SU(4)_C$ has a subgroup $SU(3)_C \times U(1)$, the U(1) of which can be identified with B-L (or its main part) in many kinds of models as in Pati-Salam model[20] or SO(10)[22]. So the spontaneous breakdown of $SU(4)_C$ to $SU(3)_C$ allows B-L violating processes like the appearance of Majorana masses of neutrinos[50] and the neutron oscillation[59] etc. In fact we showed in §3 that such a B-L violating heavy neutrino mass connected with the spontaneous breakdown of $SU(4)_C \times SU(3)_W \times U(1)_{B_0}$ really appears in the SU(7). In our SU(7) model, B-L is defined by

$$B - L = - \frac{1}{\sqrt{6}} \lambda_C^{15} - \sqrt{\frac{6}{7}}\, T_{B_0} + X ,$$

where $\lambda_C^{15}/2$ and T_{B_0} are the 15-th generator of $SU(4)_C$ and that of $U(1)_{B_0}$, respectively and X corresponds to a new $U(1)_X$ (global or local) which may be embedded with SU(7) into SO(14). The charges of X for singlet and antisymmetric tensors of fermions **1**, [2], [4] and [6] are 1, 3/7, -1/7 and -5/7, respectively.

Following the above argument, we may regard the lepton number as the fourth colour in the $SU(4)_C$. This implies that quarks of $SU(3)_C$ triplet are accompanied by leptons of $SU(3)_C$ singlet. If the $SU(4)_C$ is electrically neutral (i.e., the generator λ_C^{15} does not contribute to the charge operator), these singlets of leptons have the same charges as those of the colour triplet quarks, i.e., 2e/3 and -e/3, and were called *echo quarks* by Zee.[57] He predicted that the lightest echo quark can be observed as an absolutely stable particle. In our SU(7) with q=1/2, mirror leptons have one half of unit charge (e/2), and one of them is absolutely stable as suggested by Zee. Even in the case of q=0 where mirror leptons have normal charges, the lightest one as well as e^- can be still absolutely stable owing to no mixing between ordinary fermions and mirror ones if $U(1)_0'$ remains as an exact symmetry.

2) *Extended 'flavour'* $SU(3)_W$ *or* $SU(4)_W$

The energy region where extended 'flavour' $SU(3)_W$ or $SU(4)_W$ appears is rather severely restricted so as to produce the desired renormalization effects on the gauge coupling constants as has been shown in §5.2. We found by such a consideration that $SU(3)_C \times SU(4)_W \times U(1)$ appears in fairly high energy region ($\gtrsim 10^{12}$GeV). Therefore the $SU(4)_W$ is not attractive. Only two cases of $SU(3)_W$ are available. They are the $SU(3)_C \times SU(3)_W \times \tilde{U}(1)_q$ with q=1/2,0, and the $SU(4)_C \times SU(3)_W \times U(1)$ broken into the $SU(3)_C \times U(1)_0^{em}$ with q=0, all of which lie on the "regular" path. The fermion assignment in the $SU(3)_W$ is as follows:

3's

$$\left.\begin{array}{cccc} \left(\begin{array}{c} u \\ d \\ \hdashline D \end{array}\right)_L, & \left(\begin{array}{c} \nu_\mu \\ \mu \\ \hdashline M \end{array}\right)_L, & \left(\begin{array}{c} D^c \\ U^c \\ \hdashline u^c \end{array}\right)_L, & \left(\begin{array}{c} M^c \\ N_M^c \\ \hdashline N^c \end{array}\right)_L, \end{array}\right\} SU(2)_W$$

$\mathbf{3}^*$'s

$$\left.\begin{array}{cccc} \left(\begin{array}{c} s \\ -c \\ \hdashline C \end{array}\right)_L, & \left(\begin{array}{c} e \\ -\nu_e \\ \hdashline N_E \end{array}\right)_L, & \left(\begin{array}{c} c^c \\ -s^c \\ \hdashline s^c \end{array}\right)_L, & \left(\begin{array}{c} N_E^c \\ -E^c \\ \hdashline e^c \end{array}\right)_L, \end{array}\right\} SU(2)_W$$

1's

$$d_L^c, \quad c_L^c, \quad U_L, \quad S_L, \quad \mu_L^c, \quad N_L, \quad E_L, \quad N_{ML}.$$

The energy scale where these symmetries are broken can be as low as that of $SU(2)_W \times U(1)$, i.e., a few hundred GeV.*)

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*) In the case of the $SU(3)_C \times SU(3)_W \times \tilde{U}(1)_{1/2}$, the GUT mass becomes fairly large ($\gtrsim 10^{21}$GeV). This extraordinary large mass may be reduced by taking account of the contributions of Higgs scalars to the renormalization equations for the gauge coupling constants.

It should be noted that in the latter case ($SU(4)_C \times SU(3)_W \times U(1)$), the extended colour interaction $SU(4)_C$ also manifests itself in rather low energy region.

As far as low energy charged current processes of *ordinary* fermions are concerned, this $SU(3)_W \times U(1)$ is identical with the standard $SU(2)_W \times U(1)$ because mirror fermions are supposed to be heavier than ordinary ones. Decays of ordinary fermions mediated by the $SU(2)_W$ doublet gauge bosons among octet ones of the $SU(3)_W$, e.g., $\mu \to M + N_E + \bar{\nu}_e$, are energetically forbidden. The charged decay modes of mirror fermions, however, are rather different from those of ordinary ones. If the low energy electroweak interactions are described by the $SU(3)_W \times U(1)$, the branching ratio of the mirror fermion decay modes containing ordinary ones in the final states will become some tens percent since the $SU(2)_W$ doublet gauge bosons connect ordinary fermions and mirror ones with each other. On the other hand, if the $SU(3)_W \times U(1)$ is broken into the $SU(2)_W \times U(1)$ at much larger mass scale, such doublet gauge bosons acquire very heavy masses and mirror fermions (ordinary ones) mainly interact among themselves through the V+A (V-A) coupling for the $SU(2)_W \times U(1)$. The $SU(3)_W \times U(1)$ gauge theory introduces also an extra fitting parameter into the neutral current processes among ordinary fermions due to another neutral gauge boson coming from the $SU(3)_W$.[60] Thus the characteristics of mirror fermion decays (if they could be observed) and the re-parametrization of the neutral current processes in the shceme of $SU(3)_W \times U(1)$ will give important information to determine whether the symmetry of low energy electroweak interactions is $SU(2)_W \times U(1)$ or $SU(3)_W \times U(1)$. The present situation of experiments seems to leave room for the $SU(3)_W \times U(1)$, and searching for alternatives to the standard $SU(2)_W \times U(1)$ may not be so silly yet.

3) *Various U(1)'s*

Recently several authors analyzed the effects of additional other U(1) factors than the weak hypercharge in the neutral current processes, and especially Barr and Zee presented a general discussion in the SU(N) GUT.[56] Even though these effects would be minute, it seems premature to conclude that additional U(1) interactions exist or not.[61] Precise neutral current measurements in the future may give useful information about GUT's as stressed by Barr and Zee. Furthermore we note that these U(1)'s may be relics of extended 'colour', extended 'flavour' and/or 'horizontal' symmetries. Then, such measurements would also tell us something about the physics in the intermediate energy region.

§5.4 Further remarks

We have explored systematically the breaking patterns of symmetries in the SU(7) grand unification scheme. As mentioned in Introduction (§1), the motivations for extended GUT's are twofold; inclusion of generations and extended 'colour' and/or 'flavour' interactions. Above all, one seems so far to be inclined to look at extended GUT's from the horizontal (standard) point of view. There is, however, no inevitable connection between the inclusion of generations into GUT's and the manifestation of horizontal symmetries at low energy since the former is a group theoretical problem, while the latter may be considered to be a dynamical one. For example, horizontal symmetries may be broken at the first stage of GUT breaking not to appear clearly below the grand unification energy, and extended 'colour' and/or 'flavour' interactions rather may manifest themselves in the intermediate region. Most of the breaking patterns in extended GUT's will be equally possible within the realm of gauge theories where Higgs scalars are introduced as fundamental fields for symmetry breakings. We do not have any clear reasoning to prefer a certain path to others in the present. Then, the actual realization of a particular breaking path in our world may be simply an accident or a suggestion of some profound physics not yet known. Even though we cannot answer such a basic problem, *we believe that it is very important and meaningful to list up various possibilities and discuss them seriously because our present experimental knowledge of the particle world is restricted to "low-energy" in contrast with the huge grand unification mass.* We hope future experimental surveys beyond TeV will give us some hints about the ingenuous questions; "What is the correct grand unification group ?", "Which path is it spontaneously broken through ?", and "What kinds of new interactions appear in the intermediate region ?" One of the most promising windows through which we can have a peep at the very high-energy world is, we expect, to study very very precisely the tiny deviations from the standard $SU(2)_W \times U(1)$ theory in the neutral current processes due to extra U(1) interactions which would be relics of unknown interactions. Furthermore, the appearance of mirror fermions in the feasible future experiments is also crucial to a class of GUT's. We have given some comments on the phenomenological characteristics of mirror fermions in this section. It would be very important at present to analize rigorously the phenomenology of terrestrial mirror fermions. Effects of terrestrial mirror fermions on the flavour changing neutral current processes of ordinary fermions will be discussed in the next section.

§6. Effects of Terrestrial Mirror Fermions on Flavour Changing Neutral Currents

Effects of flavour changing neutral currents induced by mirror fermion contamination in ordinary fermions are investigated, especially on the K_L-K_S mass difference. It is shown that the effect is safely suppressed as far as "ordinary"-"mirror" mixings are sufficiently small($\lesssim 10^{-2} \sim 10^{-3}$).

§6.1 Introduction

A class of GUT's such as SU(N) and SO(N)[39)∼49)] which incorporate multigenerations introduce *mirror* fermions which have V+A coupling with respect to $SU(2)_W \times U(1)$. When we count naively the number of generations of ordinary fermions with V-A coupling following the Georgi's criterion,[36)] all the mirror fermions should drop out of our low-energy world ($\lesssim$100 GeV) by forming $SU(3)_C \times SU(2)_W \times U(1)$ invariant superheavy mass terms with additional ordinary partners through the spontaneous symmetry breakings at the early stages. Then, the smallest GUT based on a unitary group that leaves more than three generations of light fermions is SU(11).[36)] The SU(11) is, however, unattractive since it has many superheavy fermions behind and is not asymptotically free by their effects beyond the unification mass.[62)] On the other hand, the spinor representation of SO(N) ($N \geq 11$) is real with respect to the

SO(10). This implies that no fermions can survive in the orthogonal grand unification. Thus, we should invent some clever device for evasion of the "survival hypothesis" in order to incorporate multigenerations in a smaller unitary group than SU(11) or in an orthogonal group. Possible evasion mechanisms, e.g., an extra U(1), have already been discussed in §§ 3.2 and 5.2. They are essentially due to the GUT symmetry or its subsymmetry. If the grand unification including multigenerations is true, an evasion mechanism must work in such a way that may prevent mirror fermions from forming superheavy masses in cooperation with ordinary ones. Then, mirror fermions as well as ordinary ones will emerge in our low-energy world. We call such surviving fermions *"terrestrial"*, while superheavy ones *"celestial"*.

It is immediately supposed that the terrestiral mirror fermions could slightly mix with the ordinary fermions by either the Yukawa interactions with $SU(2)_W$ singlet scalars at the tree level or some radiative corrections including the superheavy particles without such $SU(2)_W$ singlet scalars, since a subsymmetry of the GUT group which separates ordinary and mirror fermions from each other should be spontaneously broken at M_W. In this section, we would like to point out that flavour changing neutral currents are induced by such mirror fermion contamination in ordinary fermions and vice versa, in spite of the fact that the neutral currents associated with the Z boson of $SU(2)_W \times U(1)$ are separately diagonal for the ordinary and mirror fermion flavours. The effects of the flavour changing neutral currents induced in this way on the K_L-K_S mass difference etc. are safely suppressed as far as "ordinary"-"mirror" mixings are sufficiently small($\lesssim 10^{-2} \sim 10^{-3}$). In what follows, we present a detailed analysis for this problem in an SU(7) grand unified model as an illustration.

§6.2 Appearance of flavour changing neutral current interactions through mirror contamination, and their suppression

We take as an anomaly free fermion combination in the SU(7)

$$\mathbf{1} + [2] + [4] + [6] \;, \tag{6·1}$$

which can be embedded into the spinor representation of SO(14) and involves two generations of ordinary fermions and the same number of mirror ones. The electirc charge is defined for the fundamental representation $\mathbf{7}$ by

$$Q = (-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, 1 \;, 0 \;, 0 \;, 0 \;). \qquad (6·2)^{*)}$$

In accordance with this charge definition, the charge spectra

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*) This charge assignment corresponds to q=0. (See §5.2.) We have here changed the order of the elements for convenience in such a way as $(Q_0, Q_1, \cdots, Q_6) \to (Q_1, \cdots Q_6, Q_0)$.

of mirror fermions become the same as those of ordinary ones. The breaking pattern in the SU(7) is

$$SU(7) \longrightarrow SU(3)_C \times SU(2)_W \times U(1) \times U(1)' \text{ at } M_X$$

$$\longrightarrow SU(3)_C \times U(1)_{em} \text{ at } M_W \, , \qquad (6\cdot 3)^{*)}$$

where U(1) and U(1)'**) represents the weak hypercharge Y/2 and an extra charge Y'/2, respectively:

$$\left.\begin{aligned} \frac{Y}{2} &= (-\tfrac{1}{3},-\tfrac{1}{3},-\tfrac{1}{3},\tfrac{1}{2},\tfrac{1}{2},0,0), \\ \frac{Y'}{2} &= (0,0,0,0,0,-\tfrac{1}{2},\tfrac{1}{2}). \end{aligned}\right\} \qquad (6\cdot 4)$$

It should be noticed that Y' does not contribute to the charge Q. Among the fermions contained in the combination (6·1), the Y' is assigned as follows:

$$\left.\begin{aligned} &Y'(u,d,\nu_e,e)=Y'(c,s,\nu_\mu,\mu)=0 && \text{for ordinary fermions,} \\ &Y'(U,D,N_E,E)=-Y'(C,S,N_M,M)=1 && \text{for mirror fermions.} \end{aligned}\right\} (6\cdot 5)$$

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*) Though there may be some intermediate stages of symmetry breaking, it is unnecessary for our discussion to consider such a complicated situation.

**) See also various U(1)'s in §5.2, where $U(1)=U(1)_0$ and $U(1)'=U(1)'_0$ corresponding to q=0.

Thus, by virtue of this extra U(1)' symmetry which remains unbroken up to M_W, the fermions can evade the Georgi's counting criterion[36)] and survive as terrestrial in our SU(7) model.

First of all, we describe the spontaneous breakdown of $SU(2)_W \times U(1) \times U(1)'$ and exhibit the generated fermion mass matrices. Then we observe that the flavour changing neutral currents associated with the Z boson are induced by diagonalizing the fermion mass matrices. The estimate of the magnitude of these flavour changing interactions will give the upper bound on the mirror fermion contamination in ordinary fermions.

As is easily seen, the last stage of symmetry breaking at M_W is caused by usual weak doublet Higgs scalars H^a (a=4,5) with Y'=0 and an extra weak singlet scalar ϕ with Y'=1. The ϕ is necessary for breaking the U(1)' symmetry and making the corresponding gauge bososn Z' massive $O(M_W)$. The mass matrix for the neutral gauge bosons W_3, B and Z' becomes as follows:

$$\begin{array}{c} \begin{array}{ccc} W_3 \quad & \quad B \quad & \quad Z' \end{array} \\ \left(\begin{array}{cc|c} g^2\tilde{v}^2 & gg'\tilde{v}^2 & 0 \\ gg'\tilde{v}^2 & g'\tilde{v}^2 & 0 \\ \hline 0 & 0 & g''\tilde{v}'^2 \end{array}\right) \end{array} , \tag{6·6}$$

where

$$\langle H^5 \rangle \equiv \tilde{v}, \qquad \langle \phi \rangle \equiv \tilde{v}' . \tag{6·7}$$

Thanks to this choice of Higgs multiplets there is no mixing between the gauge bosons of $SU(2)_W \times U(1)$ and Z' at the tree level.

As for the fermion mass generation, the H^a's produce the weak doublet mass terms which are formed among ordinary fermions and among mirror ones, respectively, while the ϕ couples ordinary fermions with mirror ones. Hereafter we restrict, for simplicity, our investigation to the first two generations of fermions which are contained in the combination (6·1). The general forms of the $SU(2)_W \times U(1) \times U(1)'$ invariant Yukawa couplings for quarks with H^a and ϕ are

$$\begin{aligned}
&\varepsilon_{ab} G^{(u)}_{\alpha\beta} \bar{u}_{\alpha R} q^a_{\beta L} H^b + \text{h.c.} + G^{(d)}_{\alpha\beta} \bar{d}_{\alpha R} q^a_{\beta L} H_a + \text{h.c.} \\
&+ \varepsilon^{ab} G^{(U)}_{\alpha\beta} \bar{Q}_{\alpha R a} U_{\beta L} H_b + \text{h.c.} + G^{(D)}_{\alpha\beta} \bar{Q}_{\alpha R a} D_{\beta L} H^a + \text{h.c.} \\
&+ F_{\alpha\beta} \bar{Q}_{\alpha R a} q^a_{\beta L} \phi + \text{h.c.} + F'_{\alpha\beta} \bar{Q}_{\alpha R a} q^a_{\beta L} \phi^* + \text{h.c.} \\
&+ F^{(u)}_{\alpha\beta} \bar{u}_{\alpha R} U_{\beta L} \phi + \text{h.c.} + F^{(u)'}_{\alpha\beta} \bar{u}_{\alpha R} U_{\beta L} \phi^* + \text{h.c.} \\
&+ F^{(d)}_{\alpha\beta} \bar{d}_{\alpha R} D_{\beta L} \phi + \text{h.c.} + F^{(d)'}_{\alpha\beta} \bar{d}_{\alpha R} D_{\beta L} \phi^* + \text{h.c.} ,
\end{aligned} \tag{6·8}$$

where $q^a_{\alpha L}$ and $Q^a_{\alpha R}$ etc. are ordinary and mirror quark doublets*)

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*) As usual, the left-handed (right-handed) ordinary fermions are $SU(2)_W$-doublet(-singlet), while the right-handed (left-handed) mirror fermions are $SU(2)_W$-doublet(-singlet).

etc., respectively:

$$\mathbf{q}_L = \begin{pmatrix} \mathbf{u} \\ \mathbf{d} \end{pmatrix}_L = \begin{pmatrix} u & c \\ d & s \end{pmatrix}_{L}, \quad \mathbf{Q}_R = \begin{pmatrix} \mathbf{U} \\ \mathbf{D} \end{pmatrix}_R = \begin{pmatrix} U & C \\ D & S \end{pmatrix}_{R}, \tag{6·9}$$

and $G^{(u)}_{\alpha\beta}$, $F^{(u)}_{\alpha\beta}$, etc. are coupling constants, indices a,b denote the $SU(2)_W$-doublet components and $\alpha,\beta=1,2$, two generations of fermions, respectively. The Yukawa couplings for leptons are expressed in similar forms. Here we give a detailed exploration only for quarks. The results for leptons will be derived immediately. From the Yukawa couplings (6·8), the mass matrices corresponding to up-type quarks and down-type ones, $\mathcal{M}^{(u)}$ and $\mathcal{M}^{(d)}$ respectively, becomes as follows:

$$\mathcal{M}^{(u)} = \begin{array}{c} \\ \bar{u}_R \\ \bar{c}_R \\ \bar{U}_R \\ \bar{C}_R \end{array} \begin{array}{c} \begin{array}{cccc} u_L & c_L & U_L & C_L \end{array} \\ \left(\begin{array}{c|c} M^{(u)}_L & M^{(u)}_{LR} \\ \hline M^{(u)}_{RL} & M^{(u)}_R \end{array} \right) \end{array} , \tag{6·10}$$

$$\mathcal{M}^{(d)} = \begin{array}{c} \\ \bar{d}_R \\ \bar{s}_R \\ \bar{D}_R \\ \bar{S}_R \end{array} \begin{array}{c} \begin{array}{cccc} d_L & s_L & D_L & S_L \end{array} \\ \left(\begin{array}{c|c} M^{(d)}_L & M^{(d)}_{LR} \\ \hline M^{(d)}_{RL} & M^{(d)}_R \end{array} \right) \end{array} . \tag{6·11}$$

Submatrices $M_L^{(u)}$ etc. are expressed in terms of the coupling constants $G_{\alpha\beta}^{(u)}$ etc. and the v.e.v.'s, $\langle H^5\rangle = \tilde{v}$ and $\langle\phi\rangle = \tilde{v}'$:

$$\left.\begin{aligned}
&(M_L^{(q)})_{\alpha\beta} = G_{\alpha\beta}^{(q)}\cdot\tilde{v}, \qquad (M_R^{(q)})_{\alpha\beta} = G_{\alpha\beta}^{(Q)}\cdot\tilde{v}, \\
&(M_{LR}^{(q)})_{\alpha\beta} = (F_{\alpha\beta}^{(q)}+F_{\alpha\beta}^{(q)\prime})\cdot\tilde{v}', \qquad (M_{RL}^{(q)})_{\alpha\beta} = (F_{\alpha\beta}+F'_{\alpha\beta})\cdot\tilde{v}', \\
&(q=u,d\ ;\ Q=U,D).
\end{aligned}\right\} \quad (6\cdot12)$$

The $\mathcal{M}^{(q)}$ is diagonalized as usual by suitable biunitary transformations:

$$O^{(q^c)\dagger}\mathcal{M}^{(q)}O^{(q)} = \mathrm{diag.}(m_{q_1}, m_{q_2}, m_{Q_1}, m_{Q_2}). \qquad (6\cdot13)$$

It is covenient to decompose the unitary matrices $O^{(q)}$ and $O^{(q^c)}$ into the following forms:

$$O^{(q)} \equiv \left(\begin{array}{c|c} O_L^{(q)} & O_{LR}^{(q)} \\ \hline O_{RL}^{(q)} & O_R^{(q)} \end{array}\right), \quad O^{(q^c)} \equiv \left(\begin{array}{c|c} O_L^{(q^c)} & O_{LR}^{(q^c)} \\ \hline O_{RL}^{(q^c)} & O_R^{(q^c)} \end{array}\right), \qquad (6\cdot14)$$

where it should be noticed that submatrices $O_L^{(q)}$ etc. are not independent each other by the unitary condition. Correspondingly, the weak interaction bases, q_α and Q_α, are suitable linear combinations of the mass eigenstates, $\tilde{q}_\alpha$ and $\tilde{Q}_\alpha$, and vice versa:

$$\left.\begin{aligned}
q_{\alpha L} &= (O_L^{(q)})_{\alpha\beta}\tilde{q}_{\beta L} + (O_{LR}^{(q)})_{\alpha\beta}\tilde{Q}_{\beta L} \,, \\
q_{\alpha R} &= (O_L^{(q^c)\dagger})_{\alpha\beta}\tilde{q}_{\beta R} + (O_{RL}^{(q^c)\dagger})_{\alpha\beta}\tilde{Q}_{\beta R} \,, \\
Q_{\alpha L} &= (O_{RL}^{(q)})_{\alpha\beta}\tilde{q}_{\beta L} + (O_R^{(q)})_{\alpha\beta}\tilde{Q}_{\beta L} \,, \\
Q_{\alpha R} &= (O_{LR}^{(q^c)\dagger})_{\alpha\beta}\tilde{q}_{\beta R} + (O_R^{(q^c)\dagger})_{\alpha\beta}\tilde{Q}_{\beta R} \,.
\end{aligned}\right\} \qquad (6\cdot 15)$$

We note that $O_{LR}^{(q)}$ and $O_{RL}^{(q^c)\dagger}$ represent the mirror fermion contamination in ordinary fermions. It is reasonable to suppose hierarchies $M_R^{(q)} \gg M_L^{(q)}$, $M_{LR}^{(q)}$, $M_{RL}^{(q)}$ (q=u,d) since mirror fermions are sufficiently heavier at least than the first two generations of ordinary ones, and "ordinary"-"mirror" mixings are expected to be rather small. Then, $O_L^{(q)}$ etc. are approximately expressed as follows:

$$\left.\begin{aligned}
O_L^{(q)} &\simeq O_L^{(q)}(0), \quad & O_L^{(q^c)} &\simeq O_L^{(q^c)}(0), \\
O_R^{(q)} &\simeq O_R^{(q)}(0), \quad & O_R^{(q^c)} &\simeq O_R^{(q^c)}(0), \\
O_{LR}^{(q)} &\simeq M_{RL}^{(q)\dagger} M_R^{(q)\dagger-1} O_R^{(q)}(0), \quad & O_{LR}^{(q^c)} &\simeq M_{LR}^{(q)} M_R^{(q)-1} O_R^{(q^c)}(0), \\
O_{RL}^{(q)} &\simeq -M_R^{(q)-1} M_{RL}^{(q)} O_L^{(q)}(0), \quad & O_{RL}^{(q^c)} &\simeq -M_R^{(q)\dagger-1} M_{LR}^{(q)} O_L^{(q^c)}(0),
\end{aligned}\right\} \qquad (6\cdot 16)$$

where the 0-th order unitary matrices $O_L^{(q)}(0)$ etc. are defined by

$$\left.\begin{aligned} O_L^{(q^c)}(0)^{\dagger} M_L^{(q)} O_L^{(q)}(0) &= \begin{pmatrix} m_{q_1} & 0 \\ 0 & m_{q_2} \end{pmatrix}, \\ O_R^{(q^c)}(0)^{\dagger} M_R^{(q)} O_R^{(q)}(0) &= \begin{pmatrix} m_{Q_1} & 0 \\ 0 & m_{Q_2} \end{pmatrix}. \end{aligned}\right\} \tag{6·17}$$

Now we investigate the flavour structure of neutral currents. As is well-known, the neutral current associated with the Z boson is given in terms of the weak interaction bases as follows:

$$\begin{aligned} J_\mu^{(Z)} &= J_\mu^{(3)} - \sin^2\theta_W J_\mu^{(em)} \\ &= \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right)(\bar{\mathbf{u}}_L\gamma_\mu\mathbf{u}_L + \bar{\mathbf{U}}_R\gamma_\mu\mathbf{U}_R) \\ &\quad - \frac{2}{3}\sin^2\theta_W\ (\bar{\mathbf{u}}_R\gamma_\mu\mathbf{u}_R + \bar{\mathbf{U}}_L\gamma_\mu\mathbf{U}_L) \\ &\quad + \left(-\frac{1}{2} + \frac{1}{3}\sin^2\theta_W\right)(\bar{\mathbf{d}}_L\gamma_\mu\mathbf{d}_L + \bar{\mathbf{D}}_R\gamma_\mu\mathbf{D}_R) \\ &\quad + \frac{1}{3}\sin^2\theta_W\ (\bar{\mathbf{d}}_R\gamma_\mu\mathbf{d}_R + \bar{\mathbf{D}}_L\gamma_\mu\mathbf{D}_L)\ . \end{aligned} \tag{6·18}$$

The $J_\mu^{(Z)}$ can be rewritten in terms of the mass eigenstates by the use of Eq.(6·15). Then, in addition to the main part obtained from Eq.(6·18) replacing $\mathbf{u}_L$, $\mathbf{U}_R$ etc. by $\tilde{\mathbf{u}}_L$, $\tilde{\mathbf{U}}_R$ etc., the following current $\tilde{J}_\mu^{(Z)}$, which involves the flavour changing currents, emerges for ordinary quarks:

$$\tilde{J}_\mu^{(Z)} = \bar{\tilde{\mathbf{u}}}_L K_L^{(u)} \gamma_\mu \tilde{\mathbf{u}}_L + \bar{\tilde{\mathbf{u}}}_R K_R^{(u)} \gamma_\mu \tilde{\mathbf{u}}_R + \bar{\tilde{\mathbf{d}}}_L K_L^{(d)} \gamma_\mu \tilde{\mathbf{d}}_L + \bar{\tilde{\mathbf{d}}}_R K_R^{(d)} \gamma_\mu \tilde{\mathbf{d}}_R \quad , \tag{6·19}$$

where

$$\left.\begin{aligned} K_L^{(u)} &= -\frac{1}{2}(O_{RL}^{(u)\dagger}\cdot O_{RL}^{(u)}) \;, & K_R^{(u)} &= \frac{1}{2}(O_{LR}^{(u^c)}\cdot O_{LR}^{(u^c)\dagger}) \;, \\ K_L^{(d)} &= \frac{1}{2}(O_{RL}^{(d)\dagger}\cdot O_{RL}^{(d)}) \;, & K_R^{(d)} &= -\frac{1}{2}(O_{LR}^{(d^c)}\cdot O_{LR}^{(d^c)\dagger}) \;. \end{aligned}\right\} \tag{6·20}$$

Since $K_L^{(u)}$ etc. are generally not diagonal matrices, we observe that flavour changing neutral currents associated with the Z boson have been induced by the mirror fermion contamination in ordinary fermions and vice versa, and thus find that the magnitude of mirror fermion contamination is directly reflected on that of their cotributions to flavour changing neutral processes such as the K_L-K_S mass difference etc.

In order to clarify this potentially dangerous situation more closely and give the order estimation of the upper bound on the mirror fermion contamination, let us investigate its effect on the K_L-K_S mass difference which is caused by the $\bar{\tilde{s}}\gamma_\mu\gamma_5\tilde{d}$ and $\bar{\tilde{d}}\gamma_\mu\gamma_5\tilde{s}$ currents. $K_L^{(d)}$ and $K_R^{(d)}$ relevant to this process are rewritten by the use of Eq.(6·16) as follows:

$$\left.\begin{aligned} K_L^{(d)} &= \frac{1}{2}\, O_L^{(d)}(0)^\dagger M_{RL}^{(d)\dagger} (M_R^{(d)} M_R^{(d)\dagger})^{-1} M_{RL}^{(d)} O_L^{(d)}(0) \ , \\ K_R^{(d)} &= -\frac{1}{2}\, M_{LR}^{(d)} (M_R^{(d)\dagger} M_R^{(d)})^{-1} M_{LR}^{(d)\dagger} \ . \end{aligned}\right\} \tag{6·21}$$

We adopt the kinship hypothesis (Eq.(6·23)) for the "ordinary"-"mirror" mixing matrices $M_{LR}^{(u)}$ etc. and suppose that $M_R^{(u)}$ and $M_R^{(d)}$ are diagonal for simplicity, i.e.,

$$M_{LR}^{(d)} \equiv v'\cdot F^{(d)} \begin{pmatrix} f & \varepsilon \\ \varepsilon' & f' \end{pmatrix} , \quad M_{RL}^{(d)} \equiv v'\cdot F^{(d)} \begin{pmatrix} f^c & \varepsilon^c \\ \varepsilon^{c'} & f^{c'} \end{pmatrix} , \tag{6·22}$$

$$\varepsilon, \varepsilon', \varepsilon^c, \varepsilon^{c'} \sim \theta_C << f, f', f^c, f^{c'} \sim O(1) \ , \tag{6·23}$$

$$M_R^{(d)} = \begin{pmatrix} m_D & 0 \\ 0 & m_S \end{pmatrix}, \text{ and therefore } O_R^{(d)}(0) = O_R^{(d^c)}(0) = \mathbf{1} \ , \tag{6·24}$$

where $F^{(d)}$ is a typical Yukawa coupling constant and θ_C is the Cabibbo angle. Furthermore, if we assume $O_L^{(u)}(0) = \mathbf{1}$, $O_L^{(d)}(0)$ becomes the Cabibbo mixing matrix:

$$O_L^{(d)}(0) = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} . \tag{6·25}$$

Substituting Eqs.(6·22)$\sim$(6·25) into Eq.(6·21), we find that the off-diagonal elements of $K_L^{(d)}$ and $K_R^{(d)}$ which correspond to $\bar{\tilde{s}}\tilde{d}$

and $\bar{\tilde{d}}\tilde{s}$ currents become

$$\left.\begin{aligned}
(K_L^{(d)})_{12}=(K_L^{(d)})_{21}&=\frac{1}{2}\,\delta^2[\sin\theta_C\cos\theta_C\{\frac{1}{m_D^2}(f^{c2}-\varepsilon^{c2})-\frac{1}{m_S^2}(f^{c'2}-\varepsilon^{c'2})\} \\
&\quad+\cos2\theta_C(\frac{f^c\varepsilon^c}{m_D^2}+\frac{f^{c'}\varepsilon^{c'}}{m_S^2})]\ , \\
(K_R^{(d)})_{12}=(K_R^{(d)})_{21}&=-\frac{1}{2}\,\delta^2[\frac{f\varepsilon'}{m_D^2}+\frac{f'\varepsilon}{m_S^2}]\ ,
\end{aligned}\right\}$$

$$\delta\equiv v'\cdot F^{(d)}\ . \tag{6·26}$$

Thus, the order of magnitude of these elements are

$$(K_L^{(d)})_{12}=(K_L^{(d)})_{21}\sim(K_R^{(d)})_{12}=(K_R^{(d)})_{21}\sim O(\theta_C\cdot(\delta/m_Q)^2)\ . \tag{6·27}$$

The ratio δ/m_Q represents the magnitude of the mirror fermion contamination in ordinary fermions since $O_{LR}^{(q)}\simeq M_{RL}^{(q)\dagger}M_R^{(q)\dagger-1}O_R^{(q)}(0)$ $\simeq O(\delta/m_Q)$ etc. In order that the contribution of $\tilde{J}_\mu^{(Z)}$ to the K_L-K_S mass difference might not alter the successful calculation by Gaillard and Lee based on the four quark scheme,[63)] we find from Eq. (6·27)

$$\kappa\cdot\theta_C^2(\delta/m_Q)^4<\frac{\alpha}{4\pi}\,\varepsilon_0\cos^2\theta_C\sin^2\theta_C\ , \tag{6·28}$$

where $\kappa\sim O(1)$ represents the effects of the ambiguous factors depending on various details of models. The experimental data implies $\varepsilon_0\simeq1.4\times10^{-3}$. Then, we obtain the order estimation of the upper bound on the mirror fermion contamination in ordinary fermions:

$$\delta/m_Q < 10^{-2}\sim 10^{-3} . \tag{6·29}$$

Ohter flavour changing processes $K_L \to e\mu$ etc. will lead to similar results.

§6.3 Concluding remarks

Finally we present a few remarks.

(i) The neutral current $J_\mu^{(Z')}$ associated with Z' of U(1)' consists of only mirror fermions in the weak interaction bases. However, $J_\mu^{(Z')}$ as well as $J_\mu^{(Z)}$ gives rise to flavour changing neutral currents of ordinary fermions in the mass eigenstate bases due to the mirror fermion contamination. This effect will be also safely suppressed as far as the condition (6·29) is satisfied.

(ii) If we choose $Y' \neq 1$ for the singlet scalar ϕ, "ordinary"-"mirror" mixings are not produced. However, when we suppose the underlying GUT interactions, such mixings $O(10^{-2}\sim 10^{-3})$ will be radiatively induced as in the SO(15) model investigated by Enqvist and Maalampi.[49] Thus, the mirror fermion contamination in ordinary fermions due to the presence of terrestrial mirror fermions are possible and interesting in a class of extended GUT's.

(iii) Of course, mirror fermion contamination effects appear in charged leptonic decays $\mu\to e\nu_\mu\bar{\nu}_e$, but the restriction on the mirror fermion contamination is not so stringent $O(<10\%)$[49),64)] from the present experimental data. We have shown that the contamination effects are rather serious $(<(1\sim 0.1)\%)$ for neutral current processes.

§7. Summary and Concluding Remarks

In the final section, we summarize the remarkable results obtained in the present work.

(I) First of all, we have investigated systematically possible structures of the gauge hierarchy of interactions in the SU(N) grand unification in §2. Taking a 'from the ground up' approach (enlarging the 'known' interactions $SU(3)_C \times SU(2)_W \times U(1)$ and seeing what emerges) and tracing the trajectories of the effective coupling constants associated with the individual subgroups of the grand unification group, we have found interesting results for the possible mass scales of new interactions in the intermediate energy region and the theoretical value of $\sin^2\theta_W$ at M_W:

(i) Several sum rules are derived, which are relations between α, α_s, $\sin^2\theta_W$ and possible threshold masses M_i of new interactions. These sum rules reveal clearly and quantitatively correlations between the enlargement of the colour group and that of the flavour one.

(ii) In the SU(N) model(I) of Dawson and Georgi, without a constraint $\kappa_1(\alpha, \alpha_s, \sin^2\theta_W)=0$ which should be satisfied in the standard SU(5), M_1 can be so small that new interactions would be revealed by feasible experiments in the TeV region if there occur *two step or more* enlargements of the gauge group between M_W and the unification scale M.

(iii) In the modified SU(N) model(II) of ours, M_1 can be as small as $10^3 \sim 10^5$GeV *even for one step case* if a suitable value for q is chosen ($0<q<1$). As the result of $q \neq 0$, the model(II) could involve new charged quarks with charge $Q \neq n/3$ or new leptons with fractional charge (n:integer). If heavy particles with such an exotic charge should be produced in e^+e^- reactions etc., this might indicate the existence of new interactions, for example, of $SU(4)_C \times SU(3)_W \times U(1)$ even below a gauge threshold M_1.

(iv) As for the value $\sin^2\theta_W$ compatible with the proton lifetime (or M), the model(I) predicts somewhat smaller value ($\sin^2\theta_W \lesssim 0.22$) independently of the number of steps, while the model(II), for example with one step, could provide the value suitable for explaining 'experimental' $\sin^2\theta_W = 0.23 \pm 0.02$.

Throughout these analyses, we have assumed for simplicity that contributions of fermions and Higgs particles can be completely eliminated ($F_1^Y = F_2^Y = F_3^Y$), higher order corrections for β-functions can be neglected and mass effects near the gauge threshold are also unimportant. Main parts of our results will, however, remain unchanged even if these effects are correctly taken into account.

(II) Secondly, we have analyzed details of an SU(7) grand unified model with a nontrivial charge assignment ($q=1/2$) in §§ 3 and 4, which has been suggested from the general considerations in §2 (model(II)), as a simple possibility to attack the important problems, i.e., the *"mystery of generation"* and the possible existence of *"oases (new interactions) in the physical desert"* (10^2GeV$\sim 10^{15}$GeV). Exploring carefully contents of the fermions in an anomaly free combination of SU(7), we have clarified attractive features of our SU(7) GUM closely connected with the choice q=1/2 (§3):

(i) The predicted value of $\sin^2\theta_W$ is quite consistent with the experimental one (0.23±0.02).

(ii) There is an oasis in the desert, i.e., a new interaction $SU(4)_C \times SU(3)_W \times U(1)$ appears in an intermediate mass scale $M_1 = 10^5 \sim 10^7$ GeV.

(iii) *Two* generations of ordinary and mirror fermions are included in the anomaly free combination of SU(7) which is naturally embedded into the spinor representation of SO(14).

(iv) Corresponding to q=1/2, the mirror fermions have exotic charges in units of 1/6 and 1/2.

(v) Therefore, the survival hypothesis is evaded for the fermions in our model by the electric charge conservation.

(vi) A neutral lepton aquires a Dirac mass through the spontaneous symmetry breaking at M_1, which is heavy enough ($O(M_1)$) to suppress ν_{eL} and $\nu_{\mu L}$ masses.

Furthermore, by analyzing the Higgs potentials as well as the masses of scalar and gauge bosons at each stage of symmetry breaking, we have found, among other things, the following two results (§4):

(i) The desired breaking path with q=1/2, $SU(7) \rightarrow SU(4)_C \times SU(3)_W \times U(1)$ at $M \rightarrow SU(3)_C \times SU(2)_W \times U(1)_{1/2}$ at $M_1 \rightarrow SU(3)_C \times U(1)^{em}_{1/2}$ at M_W, which is crucial for deriving the above mentioned features of the SU(7), is indeed realized, and

(ii) the positivity of $(\text{mass})^2$'s of physical scalars (i.e., stability of the asymmetric vacuum) under the massless condition for the scalars which cause the symmetry breaking at the successive stages is surely guaranteed, in a finite range of the coupling constants of the Higgs potentials participating in whole stages.

When we will have reached the region between several tens GeV and a few hundred GeV in the future, new type of fermions (mirror fermions) with V+A coupling for the $SU(2)_W \times U(1)$ and charge units other than 1/3, i.e., 1/2 and 1/6 might come in. We hope their appearance will prove the validity of our SU(7) model.

(III) Thirdly, we have explored systematically various breaking patterns in the SU(7) grand unification in §5. The roles of Higgs multiplets for each breaking step and the corresponding mass scale have been discussed. By referring to effective interactions appearing in the intermediate region, the breaking patterns in the SU(7) are classified into three types, i.e., "*standard*", "*regular*" and "*exotic*" ones. The characteristic features and phenomenological aspects of such new interactions have been summarized as follows:

(i) *Extended 'colour'* $SU(4)_C$: The baryon number nonconserving decays of heavy mesons which are composed of ordinary and mirror quarks are caused by the $SU(3)_C$ triplet part of the $SU(4)_C$ gauge bosons. B-L violation processes such as Majorana neutrino masses and neutrino oscillations also occur by the spontaneous breakdown of the $SU(4)_C$. As suggested by Zee, the lightest mirror lepton which belongs to an $SU(4)_C$ quartet together with ordinary quarks can be absolutely stable if there is no mixing between ordinary fermions and mirror ones due to some mechanism such as electric charge or extra U(1) charge conservation.

(ii) *Extended 'flavour' $SU(3)_W$ or $SU(4)_W$*: The $SU(4)_W$ is not attractive since the $SU(3)_C \times SU(4)_W \times U(1)$ is broken at the fairly high energy ($\gtrsim 10^{12}$GeV). In contrast with this, the energy scale where the $SU(3)_W \times U(1)$ are broken can be as low as that of $SU(2)_W \times U(1)$, i.e., a few hundred GeV. As far as low-energy charged current processes of ordinary fermions are concerned, the $SU(3)_W \times U(1)$ reproduces the same results as those of the standard $SU(2)_W \times U(1)$ since mirror fermions are supposed to be heavier than ordinary ones. The charged decay modes of mirror fermions, however, are rather different from those of ordinary ones. If the low-energy electroweak interactions are described by the $SU(3)_W \times U(1)$, the branching ratio of the mirror fermion decay modes containing ordinary ones in the final states will become some tens percent, while ordinary fermions decay dominantly into themselves. The $SU(3)_W \times U(1)$ introduces also an extra fitting parameter into the neutral current processes.

(iii) *Various U(1)'s*: Extra U(1) interactions give rise to departure from the standard $SU(2)_W \times U(1)$ in the neutral current processes. Even though these effects would be minute, it seems premature at present to conclude that extra U(1) interactions exist or not. Such extra U(1) interaction effects, if they are observed precisely, will become a window to take a peep at the very high-energy world through since they would be relics of unknown interactions which are broken at the early stages of breaking paths.

(IV) Finally, we have briefly investigated the effects of terrestrial mirror fermions appearing inevitably in a class of GUT's such as SU(N) and SO(N) which incorporate multigenerations (§6). We have shown that flavour changing neutral currents are induced if there is slight mixing between ordinary fermions and mirror ones. Therefore, the "ordinary"-"mirror" mixing must be sufficiently small ($10^{-2} \sim 10^{-3}$) in order to suppress safely the effects of the flavour changing neutral currents on the K_L-K_S mass differences etc.

Throughout this work, we have explored vigorously the possible new physics beyond the several-tens-GeV region, e.g., new gauge interactions, new type of fermions etc., based on the SU(N) grand unification, especially on the SU(7). We believe that it is very important and meaningful to list up various possibilities and discuss them seriously because our present experimental knowledge of the particle physics is restricted to the mere "low-energy" in contrast with the huge grand unification mass scale. What the true grand unification group is and which breaking pattern is actually realized are still open questions. We notice for future study that if the class of grand unified theories which include multigenerations are correct, mirror fermions will emerge in the feasible future as stressed in §6. Further investigation on the phenomenology of mirror fermions may have to be done hereafter if they really exist.

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Appendix A

Transformation properties of Higgs scalars with respect to each subgroup

1) Decomposition of SU(7) Higgs multiplets into the $SU(4)_C \times SU(3)_W \times U(1)$

$\phi^{\alpha}{}_{\beta}$: $\phi_{\tilde{g}}{}^{i}{}_{j}$ $(15,1,0)_1$, $\phi_{\tilde{X}}{}^{i}{}_{a}$ $(4,3^*,7/3)_1$, $\phi_{\tilde{W}}{}^{a}{}_{b}$ $(1,8,0)_1$, ϕ_{B_0} $(1,1,0)_1$

$$\phi_{B_0} \equiv \sqrt{\frac{7}{12}}\, \phi^{i}{}_{i}$$

$h^{\alpha\beta}$: h^{ij} $(6,1,2)_1$, h^{ia} $(4,3,-1/3)_1$, h^{ab} $(1,3^*,-8/3)_1$

$H^{\alpha\beta\gamma}{}_{\delta}$: $\tilde{H}^{ijk}{}_{l}$ $(10^*,1,2)_1$, $\tilde{H}^{aij}{}_{k}$ $(20,3,-1/3)_1$, $H^{ijk}{}_{a}$ $(4^*,3^*,5/3)_1$, $\tilde{H}^{ija}{}_{b}$ $(6^*,8,2)_1$

$\tilde{H}^{abi}{}_{j}$ $(15,3^*,-8/3)_1$, $H_i \equiv H^{456}{}_{i}$ $(4^*,1,-5)_1$, $\tilde{H}^{iab}{}_{c}$ $(4,6^*,-1/3)_1$, $T_C{}^{ij}$ $(6,1,2)_1$

$T_M{}^{ia}$ $(4,3,-1/3)_1$, $T_W{}^{ab}$ $(1,3^*,-8/3)_1$

$$T_C{}^{ij} \equiv \sqrt{\frac{5}{6}}\, H^{ijk}{}_{k}, \quad T_M{}^{ia} \equiv -\sqrt{\frac{5}{6}}\, H^{aij}{}_{j}, \quad T_W{}^{ab} \equiv \sqrt{\frac{5}{2}}\, H^{abi}{}_{i}$$

Note that $H^{abc}{}_{d} = -\frac{1}{\sqrt{5}}\varepsilon^{abc}\varepsilon_{def}T_W{}^{ef}$.

$H^{\alpha\beta}{}_{\gamma}$	:	$\tilde{H}^{ij}{}_{k}$	$\tilde{H}^{ai}{}_{j}$	$H^{ij}{}_{a}$	$\tilde{H}^{ia}{}_{b}$
		$(20,1,1)_1$	$(15,3,-4/3)_1$	$(6^*,3^*,10/3)_1$	$(4,8,1)_1$
		$H^{ab}{}_{i}$	$\tilde{H}^{ab}{}_{c}$	$T_C{}^{i}$	$T_W{}^{a}$
		$(4^*,3^*,-11/3)_1$	$(1,6^*,-4/3)_1$	$(4,1,1)_1$	$(1,3,-4/3)_1$

$$T_C{}^{i} \equiv \sqrt{\frac{2}{3}}\,H^{ij}{}_{j}, \qquad T_W{}^{a} \equiv \frac{\sqrt{3}}{2}\,H^{ak}{}_{k}$$

$h^{\alpha\beta\gamma}$	:	h^{ijk}	h^{ija}	h^{iab}	h^{456}
		$(4^*,1,3)_1$	$(6,3,2/3)_1$	$(4,3^*,-5/3)_1$	$(1,1,-4)_1$
h^{α}	:	h^{i}	h^{a}		
		$(4,1,1)_1$	$(1,3,-4/3)_1$		

$\alpha,\beta,\cdots=0\sim6$ $(SU(7))$, $i,j,\cdots=0\sim3$ $(SU(4)_C)$, $a,b,\cdots=4\sim6$ $(SU(3)_W)$.

The numbers in brackets denote the $SU(4)_C\times SU(3)_W\times U(1)$ quantum numbers, respectively. Tensors with *tilde* are traceless for the contraction of $SU(4)_C$ or $SU(3)_W$ indices.

2) Decomposition of $T_W{}^{a}$, $\tilde{H}^{ai}{}_{j}$, $\tilde{H}^{ab}{}_{c}$, h^{iab} and h^{a} into the $SU(3)_C\times SU(2)_W\times U(1)_{1/2}$

$T_W{}^{a}$	:	$T_W{}^{6}$	$T_W{}^{a'}$
		$(1,1,-1/2)_W$	$(1,2,1/2)_W$

$\tilde{H}^{ai}{}_{j}$	:	$\tilde{H}^{60}{}_{j'}$	$\tilde{H}^{a'0}{}_{j'}$	$\tilde{H}^{6i'}{}_{0}$	$\tilde{H}^{a'i'}{}_{0}$
		$(3^*,1,1/3)_W$	$(3^*,2,4/3)_W$	$(3,1,-4/3)_W$	$(3,2,-1/3)_W$
		$\approx\!\!\!\!H^{6i'}{}_{j'}$	$\approx\!\!\!\!H^{a'i'}{}_{j'}$	$\tilde{T}_W{}^{6}$	$\tilde{T}_W{}^{a'}$
		$(8,1,-1/2)_W$	$(8,2,1/2)_W$	$(1,1,-1/2)_W$	$(1,2,1/2)_W$

$$\tilde{T}_W{}^{a} \equiv \frac{2}{\sqrt{3}}\,\tilde{H}^{ai'}{}_{i'} = -\frac{1}{\sqrt{3}}(H^{a0}{}_{0} - \frac{1}{3}\,H^{ai'}{}_{i'})$$

$\tilde{H}^{ab}{}_{c}$	:	$\tilde{H}^{a'6}{}_{6}$	$\approx\!\!\!\!H^{6a'}{}_{b'}$	$\tilde{H}^{45}{}_{6}$	
		$(1,2,1/2)_W$	$(1,3,-1/2)_W$	$(1,1,3/2)_W$	
h^{iab}	:	h^{045}	$h^{0a'6}$	$h^{i'45}$	$h^{i'a'6}$
		$(1,1,3/2)_W$	$(1,2,1/2)_W$	$(3,1,2/3)_W$	$(3,2,1/3)_W$
h^{a}	:	$h^{a'}$	h^{6}		
		$(1,2,1/2)_W$	$(1,1,-1/2)_W$		

$i',j',\cdots=1\sim3$ $(SU(3)_C)$, $a',b',\cdots=4\sim5$ $(SU(2)_W)$.

The numbers in brackets denote the $SU(3)_C\times SU(2)_W\times U(1)_{1/2}$ quantum numbers, respectively. Tensors with double *tilde* are traceless for the contraction of $SU(3)_C$ or $SU(2)_W$ indices.

Appendix B

SU(7) invariant Higgs potentials

i) At the second stage

$$V_7(\mathbf{21}) = \frac{1}{2!}\cdot\frac{1}{4}[-2\nu_h^2(h^{\alpha\beta}h_{\alpha\beta})+\lambda_1(h^{\alpha\beta}h_{\alpha\beta})^2+\lambda_2 h^{\alpha\beta}h^{\gamma\delta}h_{\alpha\gamma}h_{\beta\delta}],$$

$$V_8(\mathbf{21},\mathbf{224}) = \frac{1}{2!3!}\cdot\frac{1}{2}[\lambda_3 H^{\alpha_1\alpha_2\alpha_3}{}_{\beta_1}H_{\alpha_1\alpha_2\alpha_3}{}^{\beta_1}h^{\gamma_1\gamma_2}h_{\gamma_1\gamma_2}$$

$$+\lambda_4 H^{\alpha_1\alpha_2\alpha_3}{}_{\delta}H_{\alpha_1\beta_2\beta_3}{}^{\delta}h^{\beta_2\beta_3}h_{\alpha_2\alpha_3}+\lambda_5 H^{\alpha_1\alpha_2\alpha_3}{}_{\delta}H_{\alpha_1\alpha_2\beta_3}{}^{\delta}h^{\beta_3\gamma}h_{\alpha_3\gamma}$$

$$+\lambda_6 H^{\alpha_1\alpha_2\alpha_3}{}_{\delta}H_{\alpha_1\alpha_2\alpha_3}{}^{\beta_4}h^{\delta\gamma}h_{\beta_4\gamma}+\lambda_7 H^{\alpha_1\alpha_2\alpha_3}{}_{\delta}H_{\alpha_1\alpha_2\beta_3}{}^{\beta_4}h^{\delta\beta_3}h_{\alpha_3\beta_4}$$

$$+(\lambda_8 H^{\alpha\gamma_1\gamma_2}{}_{\beta}H^{\beta\delta_1\delta_2}{}_{\alpha}h_{\gamma_1\gamma_2}h_{\delta_1\delta_2}+\text{ h.c.})$$

$$+(\lambda_9 H^{\alpha\gamma_1\gamma_2}{}_{\beta}H^{\beta\delta_1\delta_2}{}_{\alpha}h_{\gamma_1\delta_1}h_{\gamma_2\delta_2}+\text{ h.c.})],$$

$$V_9(\mathbf{224}) = \frac{1}{3!}\cdot\frac{1}{4}[-2\nu_H^2(H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\alpha_3}{}^{\alpha_4})+\lambda_{10}(H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\alpha_3}{}^{\alpha_4})^2$$

$$+\lambda_{11}H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\alpha_3}{}^{\beta_4}H^{\beta_1\beta_2\beta_3}{}_{\beta_4}H_{\beta_1\beta_2\beta_3}{}^{\alpha_4}+\lambda_{12}H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\alpha_3}{}^{\beta_4}H^{\gamma_1\gamma_2\alpha_4}{}_{\gamma_4}H_{\gamma_1\gamma_2\beta_4}{}^{\gamma_4}$$

$$+\lambda_{13}H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\beta_3}{}^{\alpha_4}H^{\gamma_1\gamma_2\beta_3}{}_{\gamma_4}H_{\gamma_1\gamma_2\alpha_3}{}^{\gamma_4}+\lambda_{14}H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\beta_3}{}^{\beta_4}H^{\gamma_1\gamma_2\alpha_4}{}_{\alpha_3}H_{\gamma_1\gamma_2\beta_4}{}^{\beta_3}$$

$$+\lambda_{15}H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\beta_3}{}^{\beta_4}H^{\gamma_1\gamma_2\beta_3}{}_{\alpha_3}H_{\gamma_1\gamma_2\beta_4}{}^{\alpha_4}+\lambda_{16}H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\alpha_2\beta_3}{}^{\beta_4}H^{\gamma_1\gamma_2\beta_3}{}_{\beta_4}H_{\gamma_1\gamma_2\alpha_3}{}^{\alpha_4}$$

$$+\lambda_{17}H^{\alpha_1\alpha_2\alpha_3}{}_{\alpha_4}H_{\alpha_1\beta_2\beta_3}{}^{\beta_4}H^{\gamma_1\beta_2\alpha_4}{}_{\alpha_3}H_{\gamma_1\alpha_2\beta_4}{}^{\beta_3}],$$

$$V_{10}(\mathbf{21,140}) = \frac{1}{2!}\Big[\frac{1}{2!}\,\alpha^{h}_{1} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\alpha_1\alpha_2}{}^{\alpha_3}(h^{\beta\gamma}h_{\beta\gamma}) + \beta^{h}_{1} H^{\alpha_1\alpha_2}{}_{\alpha_3} h_{\beta_1\gamma_2} h^{\beta_1\alpha_3} H_{\alpha_1\alpha_2}{}^{\gamma_2}$$

$$+\beta^{h}_{2} H^{\alpha_1\alpha_2}{}_{\alpha_3} h_{\beta_1\alpha_1} h^{\beta_1\beta_2} H_{\beta_2\alpha_2}{}^{\alpha_3} + \beta^{h}_{3} H^{\alpha_1\alpha_2}{}_{\alpha_3} h_{\alpha_1\gamma_2} h^{\alpha_3\beta_2} H_{\beta_2\alpha_2}{}^{\gamma_2}$$

$$+\beta^{h}_{4} H^{\alpha_1\alpha_2}{}_{\alpha_3} h_{\alpha_1\alpha_2} h^{\beta_1\beta_2} H_{\beta_1\beta_2}{}^{\alpha_3}\Big],$$

$$V_{11}(\mathbf{224,140}) = \frac{1}{2!}\Big[\frac{1}{3!}\,\alpha^{H}_{1} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\alpha_1\alpha_2}{}^{\alpha_3}(H^{\beta_1\beta_2\beta_3}{}_{\beta_4} H_{\beta_1\beta_2\beta_3}{}^{\beta_4})$$

$$+\beta^{H}_{1} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\beta_2\alpha_1}{}^{\beta_4} H^{\beta_1\beta_2\gamma_3}{}_{\alpha_2} H_{\beta_4\gamma_3}{}^{\alpha_3} + \beta^{H}_{2} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\beta_2\alpha_2}{}^{\alpha_3} H^{\beta_1\beta_2\gamma_3}{}_{\gamma_4} H_{\alpha_1\gamma_3}{}^{\gamma_4}$$

$$+(\beta^{H}_{3} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\beta_2\beta_3}{}^{\alpha_3} H^{\beta_1\beta_2\gamma_3}{}_{\alpha_2} H_{\alpha_1\gamma_3}{}^{\beta_3} + \text{h.c.}) + \beta^{H}_{4} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\beta_2\alpha_1}{}^{\beta_4} H^{\beta_1\beta_2\gamma_3}{}_{\beta_4} H_{\gamma_3\alpha_2}{}^{\alpha_3}$$

$$+\beta^{H}_{5} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\beta_2\beta_3}{}^{\alpha_3} H^{\beta_1\beta_2\beta_3}{}_{\gamma_4} H_{\alpha_1\alpha_2}{}^{\gamma_4} + \beta^{H}_{6} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\alpha_1\alpha_2}{}^{\alpha_3} H^{\beta_1\gamma_2\gamma_3}{}_{\gamma_4} H_{\gamma_2\gamma_3}{}^{\gamma_4}$$

$$+(\beta^{H}_{7} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\alpha_1\beta_3}{}^{\alpha_3} H^{\beta_1\gamma_2\gamma_3}{}_{\alpha_2} H_{\gamma_2\gamma_3}{}^{\beta_3} + \text{h.c.}) + \beta^{H}_{8} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\beta_1\alpha_1\beta_3}{}^{\beta_4} H^{\beta_1\alpha_3\gamma_2}{}_{\alpha_2} H_{\beta_4\gamma_2}{}^{\beta_3}$$

$$+\beta^{H}_{9} H^{\alpha_1\alpha_2}{}_{\alpha_3} H_{\alpha_1\alpha_2\beta_3}{}^{\beta_4} H^{\gamma_1\gamma_2\alpha_3}{}_{\beta_4} H_{\gamma_1\gamma_2}{}^{\beta_3}\Big],$$

$$V_{12}(\mathbf{21,35}) = \frac{1}{3!}\Big[\frac{1}{2!}\,\alpha^{h}_{2} h^{\alpha_1\alpha_2\alpha_3} h_{\alpha_1\alpha_2\alpha_3}(h^{\beta\gamma}h_{\beta\gamma})$$

$$+\beta^{h}_{5} h^{\alpha_1\alpha_2\alpha_3} h_{\alpha_3\beta} h^{\beta\gamma} h_{\alpha_1\alpha_2\gamma} + \beta^{h}_{6} h^{\alpha\beta_1\beta_2} h_{\beta_1\beta_2} h^{\gamma_1\gamma_2} h_{\alpha\gamma_1\gamma_2}\Big],$$

$$V_{13}(\mathbf{224,35}) = \frac{1}{3!}\Big[\frac{1}{3!}\,\alpha^{H}_{2} h^{\alpha_1\alpha_2\alpha_3} h_{\alpha_1\alpha_2\alpha_3}(H^{\beta_1\beta_2\beta_3}{}_{\beta_4} H_{\beta_1\beta_2\beta_3}{}^{\beta_4})$$

$$+\beta^{H}_{10}h^{\alpha_1\alpha_2\alpha_3}H_{\alpha_3\beta_1\beta_2}{}^{\beta_3}H^{\beta_1\beta_2\gamma_1}{}_{\beta_3}h_{\alpha_1\alpha_2\gamma_1}+\beta^{H}_{11}h^{\alpha_1\alpha_2\alpha_3}H_{\alpha_2\beta_1\beta_2}{}^{\beta_3}H^{\beta_1\beta_2\gamma_1}{}_{\alpha_3}h_{\alpha_1\beta_3\gamma_1}$$

$$+\beta^{H}_{12}h^{\alpha_1\alpha_2\alpha_3}H_{\alpha_2\alpha_3\beta_1}{}^{\beta_2}H^{\gamma_1\gamma_2\beta_1}{}_{\beta_2}h_{\alpha_1\gamma_1\gamma_2}+\beta^{H}_{13}h^{\alpha_1\alpha_2\alpha_3}H_{\alpha_1\alpha_2\beta_1}{}^{\beta_2}H^{\beta_1\gamma_1\gamma_2}{}_{\alpha_3}h_{\beta_2\gamma_1\gamma_2}$$

$$+\beta^{H}_{14}h^{\alpha_1\alpha_2\alpha_3}H_{\alpha_1\alpha_2\alpha_3}{}^{\beta_1}H^{\gamma_1\gamma_2\gamma_3}{}_{\beta_1}h_{\gamma_1\gamma_2\gamma_3}+\beta^{H}_{15}h^{\alpha_1\alpha_2\alpha_3}H_{\alpha_1\alpha_2\alpha_3}{}^{\beta_4}H^{\beta_1\beta_2\beta_3}{}_{\beta_4}h_{\beta_1\beta_2\beta_3}],\ ,$$

$$V_{14}(\mathbf{21,7}) = \frac{1}{2!}\,\alpha^{h}_{3}\ h^{\alpha}h_{\alpha}(h^{\beta\gamma}h_{\beta\gamma})+\beta^{h}_{7}\ h^{\alpha}h_{\gamma\alpha}h^{\gamma\beta}h_{\beta},$$

$$V_{15}(\mathbf{224,7}) = \frac{1}{3!}\,\alpha^{H}_{3}\ h^{\alpha}h_{\alpha}(H^{\beta\gamma\delta}{}_{\kappa}H^{\kappa}{}_{\beta\gamma\delta})+\beta^{H}_{16}\ h^{\alpha}H_{\gamma\delta\alpha}{}^{\kappa}H^{\gamma\delta\beta}{}_{\kappa}h_{\beta}$$

$$+\beta^{H}_{17}\ h^{\alpha}H_{\gamma\delta\kappa}{}^{\beta}H^{\gamma\delta\kappa}{}_{\alpha}h_{\beta}.$$

ii) At the third stage

$$V_{16}(\mathbf{140}) = \frac{1}{2!}\cdot\frac{1}{4}[-2\nu^{2}_{W1}(H^{\alpha\beta}{}_{\gamma}H_{\alpha\beta}{}^{\gamma})+\lambda^{W}_{1}(H^{\alpha\beta}{}_{\gamma}H_{\alpha\beta}{}^{\gamma})^{2}+\lambda^{W}_{2}H^{\alpha\beta}{}_{\gamma}H_{\alpha\beta}{}^{\delta}H^{\kappa\lambda}{}_{\delta}H_{\kappa\lambda}{}^{\gamma}$$

$$+\lambda^{W}_{3}H^{\alpha\beta}{}_{\gamma}H_{\alpha\beta}{}^{\delta}H^{\kappa\gamma}{}_{\lambda}H_{\kappa\delta}{}^{\lambda}+\lambda^{W}_{4}H^{\alpha\delta}{}_{\beta}H_{\alpha\gamma}{}^{\beta}H^{\kappa\gamma}{}_{\lambda}H_{\kappa\delta}{}^{\lambda}+\lambda^{W}_{5}H^{\alpha\gamma}{}_{\delta}H_{\gamma\varepsilon}{}^{\beta}H^{\kappa\delta}{}_{\alpha}H_{\kappa\beta}{}^{\varepsilon}$$

$$+\lambda^{W}_{6}H^{\alpha\gamma}{}_{\delta}H_{\gamma\varepsilon}{}^{\beta}H^{\kappa\varepsilon}{}_{\alpha}H_{\kappa\beta}{}^{\delta}+\lambda^{W}_{7}H^{\alpha\gamma}{}_{\delta}H_{\gamma\varepsilon}{}^{\beta}H^{\kappa\varepsilon}{}_{\beta}H_{\kappa\alpha}{}^{\delta}],$$

$$V_{17}(\mathbf{35}) = \frac{1}{3!}\cdot\frac{1}{4}[-2\nu^{2}_{W2}(h^{\alpha\beta\gamma}h_{\alpha\beta\gamma})+\lambda^{W}_{8}(h^{\alpha\beta\gamma}h_{\alpha\beta\gamma})^{2}$$

$$+\lambda^{W}_{9}h^{\alpha\beta\gamma}h_{\delta\varepsilon\gamma}h^{\delta\varepsilon\kappa}h_{\alpha\beta\kappa}],$$

$$V_{18}(\mathbf{7}) = -\ \frac{1}{2}\ \nu^{2}_{W3}(h^{\alpha}h_{\alpha})+\ \frac{1}{4}\lambda^{W}_{10}(h^{\alpha}h_{\alpha})^{2},$$

$$V_{19}(\mathbf{140,35}) = \frac{1}{2!3!}\cdot\frac{1}{2}[\lambda^W_{11}H^{\alpha\beta}{}_\gamma H_{\alpha\beta}{}^\gamma h^{\delta\varepsilon\kappa}h_{\delta\varepsilon\kappa}+\lambda^W_{12}H^{\alpha\beta}{}_\gamma h_{\delta\varepsilon\beta}h^{\delta\varepsilon\kappa}H_{\alpha\kappa}{}^\gamma$$

$$+\lambda^W_{13}H^{\alpha\beta}{}_\gamma h_{\delta\varepsilon\kappa}h^{\gamma\varepsilon\kappa}H_{\alpha\beta}{}^\delta+\lambda^W_{14}H^{\alpha\beta}{}_\gamma h_{\alpha\beta\delta}h^{\gamma\varepsilon\kappa}H_{\varepsilon\kappa}{}^\delta$$

$$+\lambda^W_{15}H^{\alpha\beta}{}_\gamma h_{\beta\delta\varepsilon}h^{\gamma\kappa\varepsilon}H_{\alpha\kappa}{}^\delta+\lambda^W_{16}H^{\alpha\beta}{}_\gamma h_{\alpha\beta\delta}h^{\gamma\varepsilon\kappa}H_{\varepsilon\kappa}{}^\delta],$$

$$V_{20}(\mathbf{140,7}) = \frac{1}{2!}\cdot\frac{1}{2}[\lambda^W_{17}H^{\alpha\beta}{}_\gamma H_{\alpha\beta}{}^\gamma h^\delta h_\delta+\lambda^W_{18}H^{\alpha\beta}{}_\gamma h_\alpha h^\delta H_{\delta\beta}{}^\gamma+\lambda^W_{19}H^{\alpha\beta}{}_\gamma h_\delta h^\gamma H_{\alpha\beta}{}^\delta],$$

$$V_{21}(\mathbf{35,7}) = \frac{1}{3!}\cdot\frac{1}{2}[\lambda^W_{20}h^{\alpha\beta\gamma}h_{\alpha\beta\gamma}h^\delta h_\delta+\lambda^W_{21}h^{\alpha\beta\gamma}h_\gamma h^\delta h_{\alpha\beta\delta}].$$

Here we have assumed the invariance under discrete transformations:

$$\phi^\alpha{}_\beta \to -\phi^\alpha{}_\beta,\quad h^{\alpha\beta} \to -h^{\alpha\beta},\quad H^{\alpha\beta\gamma}{}_\delta \to -H^{\alpha\beta\gamma}{}_\delta,$$

$$H^{\alpha\beta}{}_\gamma \to -H^{\alpha\beta}{}_\gamma,\quad h^{\alpha\beta\gamma} \to -h^{\alpha\beta\gamma},\quad h^\alpha \to -h^\alpha.$$

The coupling constants in $\tilde{V}_{eff}$ are expressed in terms of those in $V_7+V_8+V_9$ as follows:

$$\tilde{\lambda}_1 = 2\lambda_1,\quad \tilde{\lambda}_2 = \lambda_2,\quad \tilde{\lambda}_3 = 6\lambda_{10},$$

$$\tilde{\lambda}_4 = \lambda_3 + \frac{1}{6}\lambda_5,\quad \tilde{\lambda}_5 = \frac{1}{2}\lambda_6 - \frac{1}{6}\lambda_7.$$

Appendix C

The masses of Higgs scalars at the second stage

$\tilde{H}^{ai}_{j}$

	$\tilde{\alpha}_1$	β^h_1	β^h_2	β^h_3	β^h_4	$\tilde{\beta}^H_1$	$\tilde{\beta}^H_2$	$\tilde{\beta}^H_4$	$\tilde{\beta}^H_5$	
$m^2_{\tilde{H}}{}^{60}_{j'}$	$=(1$	0	1	0	2	-1	0	1	$0)$	v^2_1
$m^2_{\tilde{H}}{}^{a'0}_{j'}$	$=(1$	0	$\frac{1}{2}$	0	0	0	0	1	$0)$	v^2_1
$m^2_{\tilde{H}}{}^{6i'}_{0}$	$=(1$	1	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	1	1	$6)$	v^2_1
$m^2_{\tilde{H}}{}^{a'i'}_{0}$	$=(1$	1	0	0	0	0	1	1	$6)$	v^2_1
$m^2_{\tilde{\tilde{H}}}{}^{6i'}_{j'}$	$=(1$	0	$\frac{1}{2}$	0	0	0	0	1	$0)$	v^2_1
$m^2_{\tilde{\tilde{H}}}{}^{a'i'}_{j'}$	$=(1$	0	0	0	0	0	0	1	$0)$	v^2_1

$\tilde{H}^{ab}_{c}$

	$\tilde{\alpha}_1$	β^h_1	β^h_2	$\tilde{\beta}^H_4$	$\tilde{\beta}^H_9$	
$m^2_{\tilde{\tilde{H}}}{}^{6a'}_{b'}$	$=(1$	0	$\frac{1}{2}$	2	$2)$	v^2_1
$m^2_{\tilde{H}}{}^{45}_{6}$	$=(1$	1	0	2	$2)$	v^2_1

$T_W{}^{a'} - \tilde{T}_W{}^{a'} - \tilde{H}^{a'6}{}_{6}$

M_{AB}	$\tilde{\alpha}_1$	β^h_1	β^h_2	β^h_3	$\tilde{\beta}^H_2$	$\tilde{\beta}^H_3$	$\tilde{\beta}^H_4$	$\tilde{\beta}^H_5$	$\tilde{\beta}^H_7$	$\tilde{\beta}^H_9$	
M_{11}	$=(1$	$\frac{10}{24}$	$\frac{5}{24}$	$-\frac{1}{6}$	$\frac{1}{12}$	$-\frac{2}{3}$	$\frac{5}{3}$	$\frac{1}{2}$	$-\frac{2}{3}$	$-\frac{4}{3})$	v^2_1
$M_{12}=M_{21}$	$=(0$	$-\frac{1}{4}$	$-\frac{1}{8}$	$\frac{1}{4}$	$-\frac{1}{4}$	1	0	$-\frac{3}{2}$	1	$0)$	v^2_1
$M_{13}=M_{31}$	$=(0$	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{2\sqrt{6}}$	$\frac{1}{4\sqrt{6}}$	0	0	0	0	0	$0)$	v^2_1
M_{22}	$=(1$	$\frac{3}{4}$	$\frac{3}{8}$	0	$\frac{3}{4}$	0	1	$\frac{9}{2}$	0	$0)$	v^2_1
$M_{23}=M_{32}$	$=(0$	0	0	$-\frac{\sqrt{6}}{8}$	0	0	0	0	0	$0)$	v^2_1
M_{33}	$=(1$	$\frac{1}{2}$	$\frac{1}{4}$	0	0	0	2	0	0	$2)$	v^2_1

M_{AB} are the matrix elements of $(\text{mass})^2$ for the $SU(2)_W$ doublets.

The indices 1,2,3, refer to ${T_W}^{a'}$, ${\tilde{T}_W}^{a'}$ and $\tilde{H}^{a'6}{}_{6'}$, respectively. Note that $2M_{33} = m^2_{\tilde{\tilde{H}}6a'}{}_{b'} + m^2_{\tilde{H}45}{}_{6}$ as pointed in §4.3.

${T_W}^6 - {\tilde{T}_W}^6$

M^6_{AB} :	$\tilde{\alpha}_1$	$(\beta^h_1 - \frac{1}{2}\beta^h_3 + 2\beta^h_4)$	β^h_2	$(\tilde{\beta}^H_1 - \tilde{\beta}^H_2 - 6\tilde{\beta}^H_5)$	$\tilde{\beta}^H_4$	$\tilde{\beta}^H_7$	$\tilde{\beta}^H_9$	
M^6_{11}	$=(1$	$\frac{1}{12}$	$\frac{13}{24}$	$-\frac{1}{12}$	$\frac{5}{3}$	$-\frac{2}{3}$	$-\frac{4}{3})$	v^2_1
$M^6_{12} = M_{21}$	$=(0$	$-\frac{1}{2}$	$-\frac{1}{8}$	$\frac{1}{2}$	0	1	$0)$	v^2_1
M^6_{22}	$=(1$	$\frac{3}{4}$	$\frac{7}{8}$	$-\frac{3}{4}$	1	0	$0)$	v^2_1

M^6_{AB} are the matrix elements of $(\mathrm{mass})^2$ for the $SU(2)_W$ singlets. The indices 1,2 refer to ${T_W}^6$ and ${\tilde{T}_W}^6$, respectively.

h^{iab} :	$\tilde{\alpha}_2$	β^h_5	β^h_6	$(\tilde{\beta}^H_{10} + \tilde{\beta}^H_{12})$	$(\tilde{\beta}^H_{11} - \tilde{\beta}^H_{13})$	
$m^2_h 045$	$=(1$	$-\frac{1}{3}$	0	$\frac{2}{3}$	$-\frac{2}{3}$	$)v^2_1$
$m^2_h 0a'6$	$=(1$	$-\frac{2}{3}$	1	$\frac{2}{3}$	$-\frac{2}{3}$	$)v^2_1$
$m^2_h i'45$	$=(1$	0	0	$\frac{2}{3}$	0	$)v^2_1$
$m^2_h i'a'6$	$=(1$	$-\frac{1}{3}$	0	$\frac{2}{3}$	0	$)v^2_1$

h^a :	$\tilde{\alpha}_3$	β^h_7	$\tilde{\beta}^H_{16}$	
$m^2_h a'$	$=(1$	0	$\frac{1}{3}$	$)v^2_1$
$m^2_h 6$	$=(1$	$\frac{1}{2}$	$\frac{1}{3}$	$)v^2_1$

In the above we have used definitions

$$\tilde{\alpha}_i \equiv \alpha^h_i + \alpha^H_i \cdot \frac{v'^2_1}{v^2_1}, \qquad \tilde{\beta}^H_i \equiv \beta^H_i \cdot \frac{v'^2_1}{v^2_1}.$$

Appendix D

The masses of gauge bosons

Some of the gauge bosons aquire masses through the minimal couplings with Higgs multiplets when symmetries are spontaneously broken down. The local SU(7) invariant kinetic term of a irreducible representation of Higgs multiplet h^A is

$$\frac{1}{N}(\mathbf{D}^\mu\cdot\mathbf{h})^A(\mathbf{D}_\mu\cdot\mathbf{h})_A \ , \tag{D·1}$$

where A denotes a set of SU(7) indices and N is a suitable normalization factor (e.g., N=2! for $h^{\alpha\beta}$(**21**) due to the duplication of $h^{12}h_{12}$, for example, in the sum of $h^{\alpha\beta}h_{\alpha\beta}$). The covariant derivative $\mathbf{D}_\mu$ is expressed by

$$(\mathbf{D}_\mu)^A{}_B = \delta^A{}_B\partial_\mu - ig\sum_{a=1}^{48} A^a_\mu(L^a)^A{}_B \ , \tag{D·2}$$

where A^a_μ is the gauge boson and L^a is the representation matrix of the corresponding generator. The explicit forms of $(L^a)^A{}_B$ for $h^{\alpha_1\cdots\alpha_n}$ and $H^{\alpha_1\cdots\alpha_n}{}_\beta$ (totally antisymmetrized with respect to $\alpha_1\sim\alpha_n$ and traceless for upper and lower indices), which are available for our analysis, are given by

$$(L^a)^{\alpha_1\cdots\alpha_r}{}_{;\alpha'_1\cdots\alpha'_r} = (\frac{\lambda^a}{2})^{\alpha_1}{}_{\alpha'_1}\cdot\delta^{\alpha_2}{}_{\alpha'_2}\cdots\delta^{\alpha_r}{}_{\alpha'_r} + \cdots$$

$$+\delta^{\alpha_1}{}_{\alpha'_1}\cdots\delta^{\alpha_{r-1}}{}_{\alpha'_{r-1}}\cdot(\frac{\lambda^a}{2})^{\alpha_r}{}_{\alpha'_r} \tag{D·3}$$

for $h^{\alpha_1 \cdots \alpha_r}$ and

$$(L^a)^{\alpha_1 \cdots \alpha_r}{}_{\beta;\alpha'_1 \cdots \alpha'_r}{}^{\beta'} = (L^a)^{\alpha_1 \cdots \alpha_r}{}_{;\alpha'_1 \cdots \alpha'_r} \cdot \delta_\beta{}^{\beta'} - \delta^{\alpha_1}{}_{\alpha'_1} \cdots \delta^{\alpha_r}{}_{\alpha'_r} \cdot (\frac{\lambda^{a^T}}{2})_\beta{}^{\beta'} \tag{D·4}$$

for $H^{\alpha_1 \cdots \alpha_n}{}_\beta$, respectively where $\lambda^a/2$ is the hermitian matrix corresponding to the generator of the fundamental representation of SU(7). Replacing h^A by its v.e.v., $v^A (=<h^A>)$, in Eq.(D·1), we obtain the $(\text{mass})^2$ matrix $\mathcal{M}$ of the gauge bosons:

$$\mathcal{M}^{ab} = \frac{1}{N} g^2 [(x^a)^A (x^b)_A + (x^b)^A (x^a)_A], \tag{D·5}$$

where

$$(x^a)^A \equiv (L^a)^A{}_B v^B$$

$$= (L^a)^{\alpha_1 \cdots \alpha_m}{}_{\beta_1 \cdots \beta_n ; \alpha'_1 \cdots \alpha'_m}{}^{\beta'_1 \cdots \beta'_n} \cdot v^{\alpha'_1 \cdots \alpha'_m}{}_{\beta'_1 \cdots \beta'_n} . \tag{D·6}$$

If we take, for example, a regular representation $\phi^\alpha{}_\beta$ (N=2 because $\phi^\alpha{}_\beta$ is a real representation), we have

$$\left.\begin{aligned} (L^a)^\alpha{}_{\beta;\alpha'}{}^{\beta'} &= (\frac{\lambda^a}{2})^\alpha{}_{\alpha'} \delta_\beta{}^{\beta'} - \delta^\alpha{}_{\alpha'} (\frac{\lambda^{a^T}}{2})_\beta{}^{\beta'} , \\ (x^a)^\alpha{}_\beta &= \frac{1}{2} [\lambda^a, <\phi>]^\alpha{}_\beta , \\ \mathcal{M}^{ab} &= -\frac{1}{4} g^2 \, \mathrm{Tr}\{[\lambda^a, <\phi>] \; [\lambda^b, <\phi>]\} . \end{aligned}\right\} \tag{D·7}$$

On the other hand, the Goldstone modes of m Higgs multiplets $h_1^{A_1}, \cdots, h_m^{A_m}$ get mass terms by introducing the gauge fixing term $-(1/2)F^aF^a$ in the R_ξ-gauge:

$$-\frac{1}{2} F^aF^a \to - \sum_{i,j} h'_{iA_i} [\frac{2g^2}{\xi N_i N_j} \sum_a (x_i^a)^{A_i} (x_j^a)_{B_j}] h'^{B_j}_j , \qquad (D\cdot 8)$$

$$(i,j = 1 \sim m)$$

where

$$F^a \equiv \sqrt{\xi} [\partial^\mu A_\mu^a - \frac{2ig}{\xi} \sum_i \{ \frac{1}{N_i} v_i^{A_i} (L_i^a \cdot h'_i)_{A_i} \}] , \qquad h'^{A_i}_i \equiv h_i^{A_i} - v_i^{A_i} . \qquad (D\cdot 9)$$

Then the $(\text{mass})^2$ matrix elements of the Goldstone modes are

$$(\mathcal{M}_{GS})_{IJ} = - \sum_{i,j} \frac{2P^i_{IA_i} \cdot P_J^{jB_j}}{\xi N_i N_j} g^2 \sum_a (x_i^a)^{A_i} (x_j^a)_{B_j} , \qquad (D\cdot 10)$$

where $P^i_{IA_i}$ is the projection operator of $h'^{A_i}_i$ into the Goldstone modes h_I. Of course the eigenvalues of $\mathcal{M}_{GS}$ and those of $\mathcal{M}^{ab}$ are identical. For example, $\mathcal{M}_{GS}$ $(\xi=1)$ for $(\text{Im } h^{06}, \text{Im } H^0)$ in the second stage becomes as follows:

$$\mathcal{M}_{GS} = \frac{2}{7} g^2 \begin{pmatrix} 5v_1^2 & 2v_1v'_1 \\ 2v_1v'_1 & 12v'^2_1 \end{pmatrix} . \qquad (D\cdot 11)$$

By diagonalizing this $(\text{mass})^2$ matrix, we obtain Eq.(4·55) in §4.3.

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